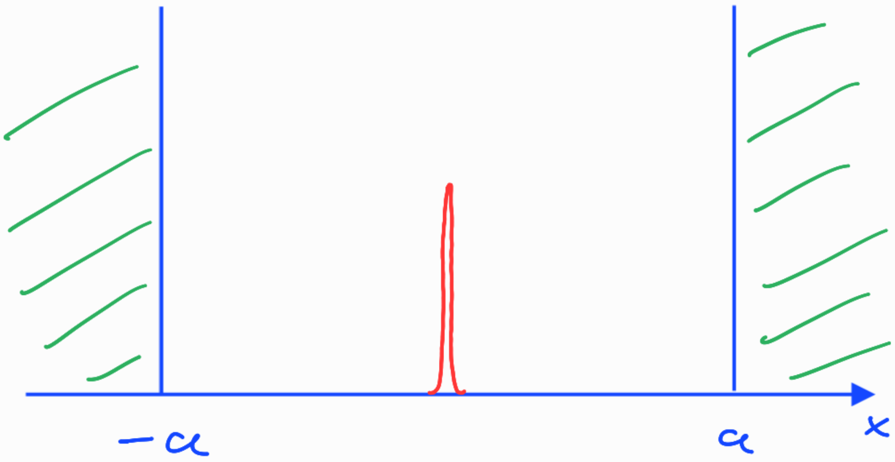
Letzte Vorlsg.: Doppelkastenpotenzial



$$u(x) = \begin{cases} \infty & : |x| > a \\ \underline{u\,\delta(x)} & : |x| \le a \end{cases}$$

st. S.Gl.:

$$\psi_E''(x) = \frac{2m}{\hbar^2}\left(u\,\delta(x) - E\right)\psi_E(x)$$

+ Randbed.: $\psi_E(-a) = \psi_E(a) = 0$

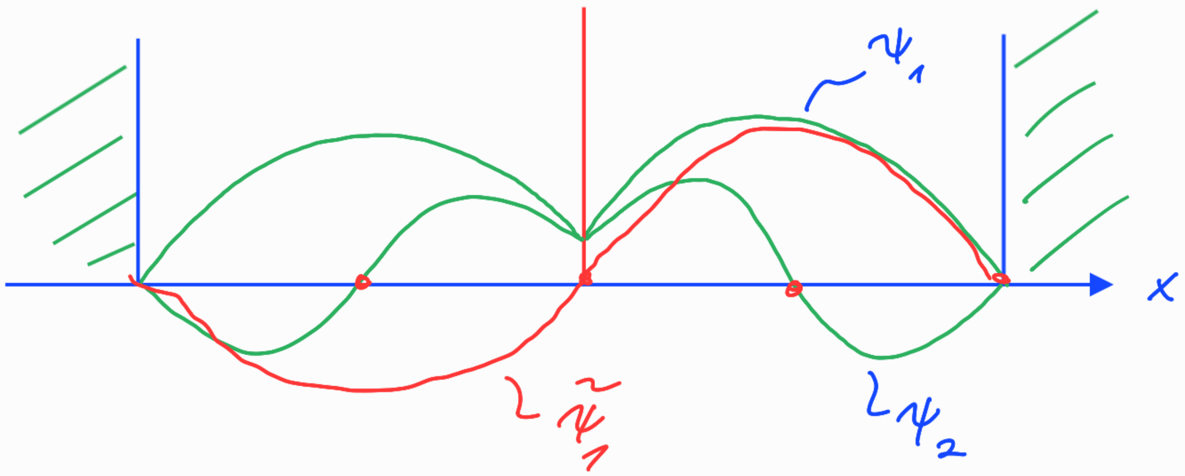
+ Anschlussbed.: $\psi_E'(0+) - \psi_E'(0-) \overset{!}{=} \frac{2mu}{\hbar^2}\psi(0)$

→ norm. Energieeigenzust. (= gebundene Zustände)

$$\psi_n(x) = c_n \sin\left(k_n\left(|x| + b\right)\right)$$

$$\boxed{b = \frac{\hbar^2}{mu}} \qquad (\ll a)$$

$$k_n = \frac{\pi}{a+b}\cdot n \,, \qquad E_n = \frac{\hbar^2}{2m}\,\frac{\pi}{(a+b)^2}\cdot n^2$$



weitere Energieeigenzustände ?

antisymm. Wellenfunktion ungeraden Indexes!

$$\Rightarrow \boxed{\begin{aligned} \tilde{\psi}_n(x) &= \frac{1}{\sqrt{a}} \sin \tilde{k}_n x \\ \tilde{k}_n &= \frac{\pi}{a} \cdot n \\ \tilde{E}_n &= \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 \pi^2}{2m} \frac{n^2}{a^2} \end{aligned}}$$

- Lsg. von st. SG. für $x \neq 0$ ✓
- $\tilde{\psi}_n(-a) = \tilde{\psi}_n(a) = 0$ ✓
- Anschlussbed.: $0 \doteq \tilde{\psi}_n'(0+) - \tilde{\psi}_n'(0-) = \frac{2m\alpha}{\hbar^2} \underbrace{\tilde{\psi}_n(0)}_{\overset{!}{=}0}$ ✓ !

L

Energieeigenfkt. $\psi_n, \tilde{\psi}_n$ zu Energien

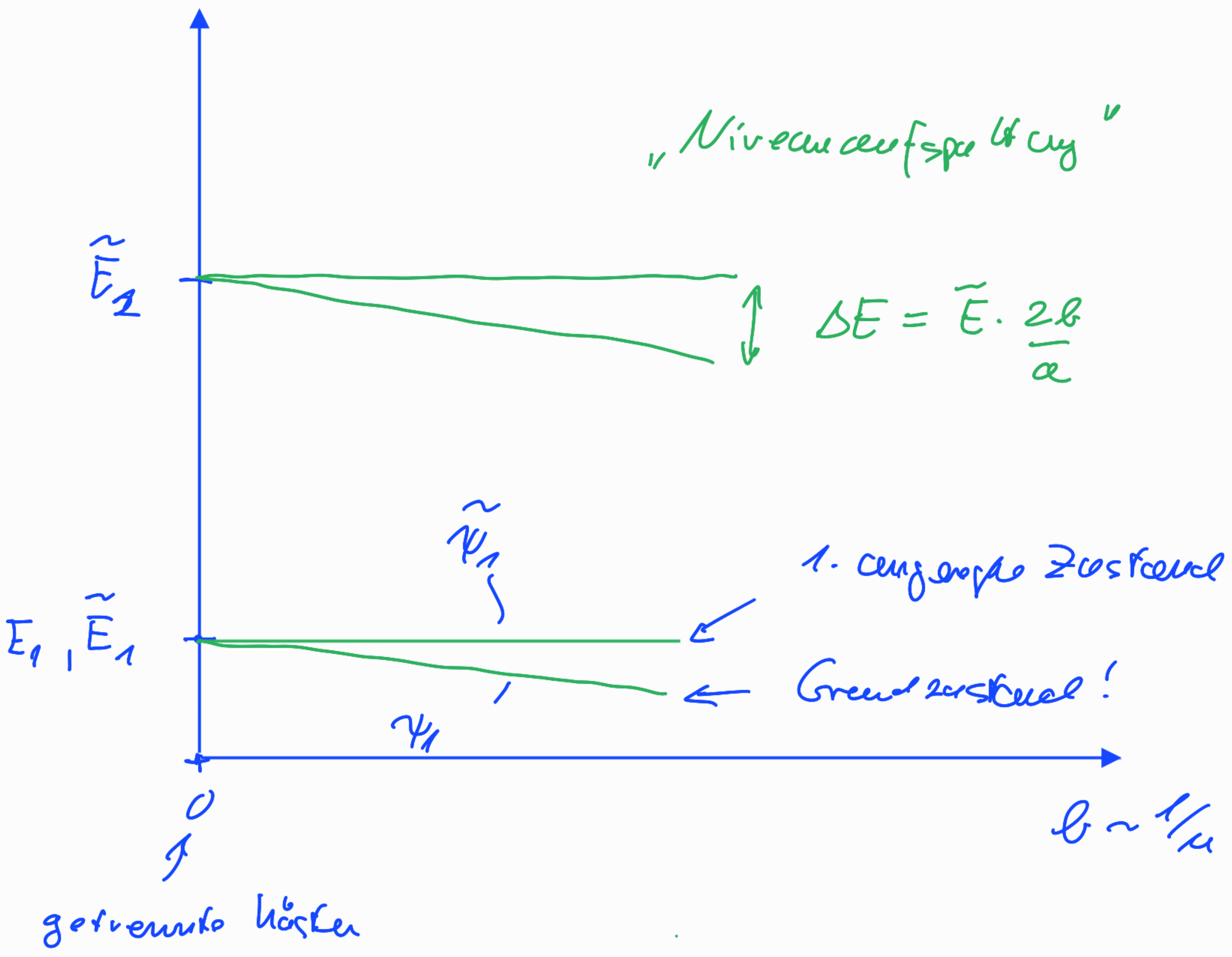
$$E_n = \frac{\hbar^2 \pi^2}{2m} \frac{n^2}{(a+b)^2} \quad , \quad \tilde{E}_n = \frac{\hbar^2 \pi^2}{2m} \frac{n^2}{a^2}$$

$$b = \frac{\hbar^2}{mu} \sim \frac{1}{u}$$

$$b \ll a$$

$$\Delta E_n = \tilde{E}_n - E_n = \tilde{E}_n \Bigg(1 - \underbrace{\frac{1}{\left(1+\frac{b}{a}\right)^2}}_{1 - 2b/a} \Bigg)$$

$$\Delta E_n = \tilde{E}_n \cdot \frac{2b}{a} \ll \tilde{E}_n$$



physikalische Signifikanz von $\Delta E_n \ll \tilde{E}_n \approx E_n$?

$$\frac{1}{\mu} \sim \frac{\Delta E_n}{\hbar} = \underline{\underline{\omega_n}} \ll \underline{\underline{\Omega_n}} = \frac{\tilde{E}_n}{\hbar}$$

Oszillationen zwischen den Kästen!

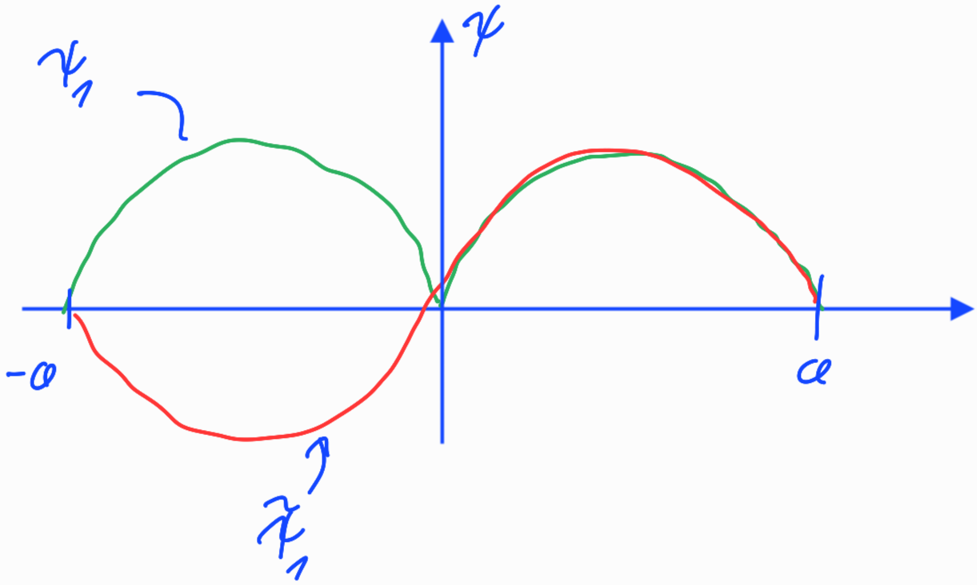
Oszillationen in einem Kasten

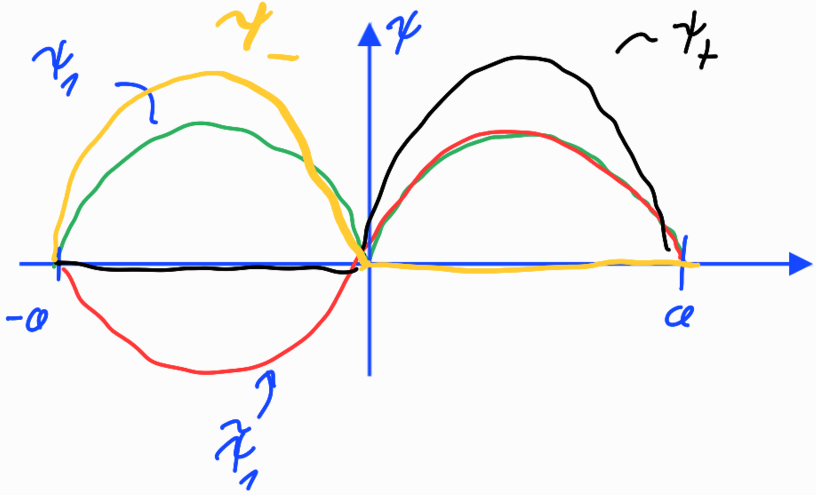
Dynamik im Doppelkasten: Näherung: $\underline{\underline{b \ll a}}$

$$\to \quad \tilde{\psi}_1(x) = \frac{1}{\sqrt{a}} \sin \frac{\pi}{a} x$$

$$\psi_1(x) = \frac{1}{\sqrt{a'}} \sin \frac{\pi}{a+b} (|x| + b) \simeq \frac{1}{\sqrt{a}} \sin \frac{\pi}{a} |x|$$

$$a' = a + O(b)$$





Betrachte Zustände:

$$|\psi_\pm\rangle = \left(|\psi_1\rangle \pm |\tilde\psi_1\rangle\right)/\sqrt{2}$$

$$\hookrightarrow \psi_\pm(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) \pm \tilde\psi_1(x)\right) =$$

$|\psi_\pm\rangle \mathrel{\hat=}$ „Teilchen im $\left\{\begin{matrix}\text{rechten}\\ \text{linken}\end{matrix}\right\}$ Kasten"

Dynamik für $|\psi(t)\rangle \in \mathrm{Span}\{|\psi_1\rangle, |\tilde\psi_1\rangle\}$

$$|\psi_1\rangle \mathrel{\hat=} \begin{pmatrix}1\\0\end{pmatrix} \qquad |\tilde\psi_1\rangle \mathrel{\hat=} \begin{pmatrix}0\\1\end{pmatrix}$$

$\Rightarrow t=0: \quad |\psi(0)\rangle = |\psi_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$$t \hookrightarrow |\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-iE_1 t/\hbar}|\psi_1\rangle + e^{-i\tilde E_1 t/\hbar}|\tilde\psi_1\rangle\right)$$

$\Omega = \Omega_1$

$\omega = \omega_1$

$$= \frac{e^{-i\Omega t}}{\sqrt{2}}\left(e^{i\omega t}|\psi_1\rangle + |\tilde\psi_1\rangle\right) = \frac{e^{-i\Omega t}}{\sqrt{2}}\begin{pmatrix}e^{i\omega t}\\ 1\end{pmatrix}$$

$$\psi_+(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin \frac{\pi}{a} x \\ 0 & : x < 0 \end{cases}$$

$$\psi_-(x) = \begin{cases} 0 & : x > 0 \\ -\sqrt{\frac{2}{a}} \sin \frac{\pi}{a} x & : x < 0 \end{cases}$$

$$|\psi(0)\rangle = |\psi_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} .$$

$$\downarrow t$$

$$|\psi(t)\rangle = \frac{e^{-i\omega t}}{\sqrt{2}} \begin{pmatrix} e^{i\omega t} \\ 1 \end{pmatrix} .$$

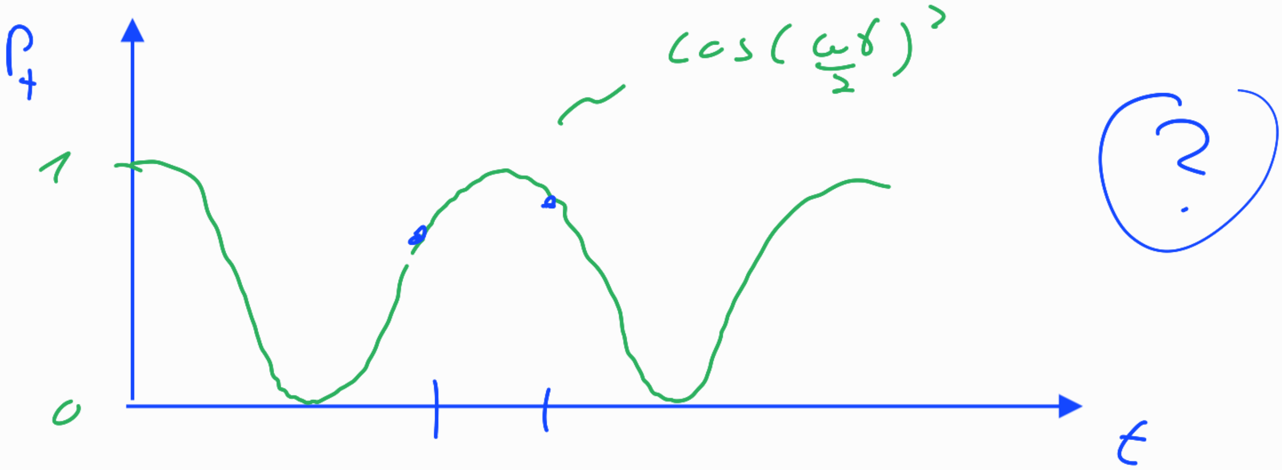
→ Wkt, dass Teilchen im rechten Kasten

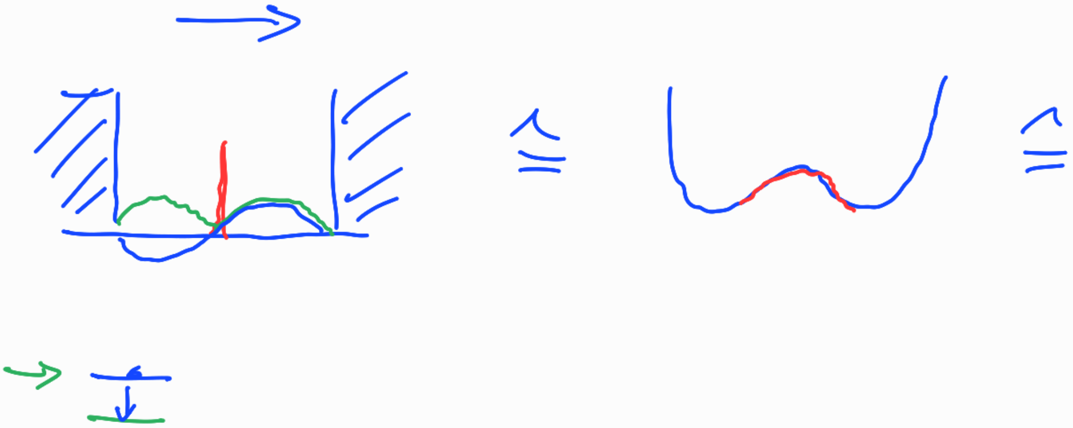
zur Zeit $t > 0$:

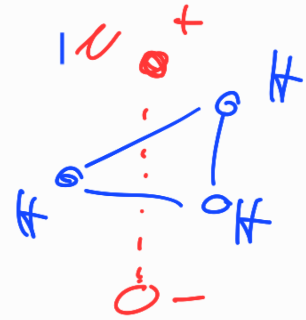
$$P_+(t) = \left|\langle \psi_+ | \psi(t)\rangle\right|^2 = \frac{1}{4} \left|\left(e^{i\omega t} + 1\right) e^{-i\omega t}\right|^2$$

$$= \frac{1}{4}\left(2 + 2\cos\omega t\right) = \frac{1}{2}\left(1 + \cos\omega t\right)$$

$$= \left(\cos\frac{\omega t}{2}\right)^2 , \qquad \omega = \frac{\Delta E_1}{\hbar} !$$





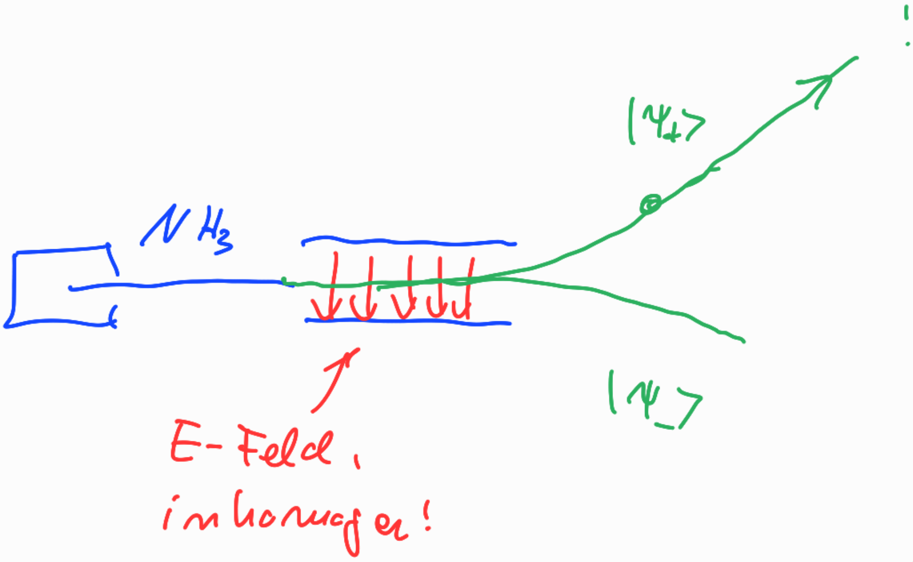


$\underline{\underline{NH_3}}$: $\frac{\omega}{2} = \underline{\underline{24\,GHz}}$

induziertes el. Dipolmoment: $\vec{\mu}$

$\rightarrow \mu_x$

$$\mu_x = \mu_0 \left(|\psi_+\rangle\langle\psi_+| - |\psi_-\rangle\langle\psi_-| \right)$$



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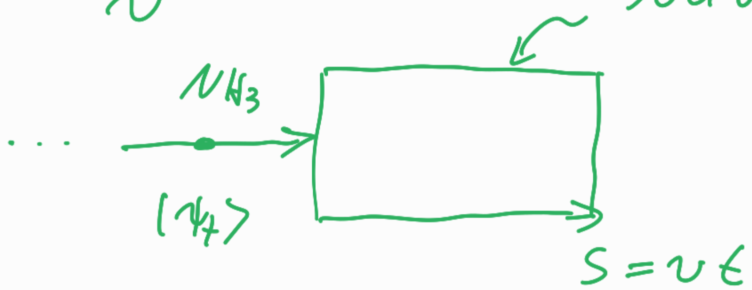
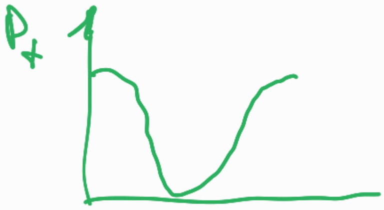


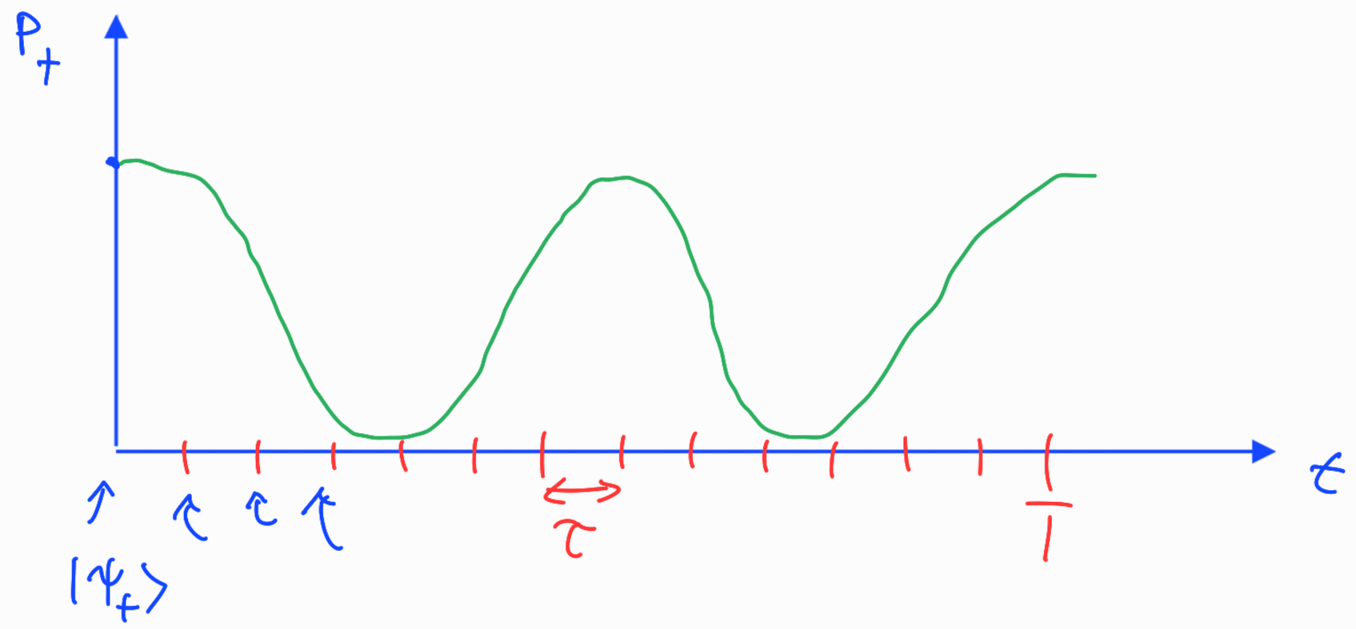


→ ... → Mikrowellen Verstärkung

durch stimulierte Emission

von Strahlung

MASER : Townes 1953 !



$\rightarrow$ $N = T/\tau$ <u>ideale</u> Messungen ($|\psi_+\rangle\langle\psi_+|$)

Wkt, dass alle N Messungen positives Ergebnis liefern?

$$\underline{P_N} = \left(\left| \langle \psi_+ | \psi(\tau) \rangle \right|^2 \right)^N \qquad \tau = T/N$$

$$= \left(\frac{1}{2} \left(1 + \underbrace{\cos \omega\tau}_{1 - \frac{1}{2}(\omega\tau)^2} \right) \right)^N$$

$$= \left(\underbrace{1 - \frac{1}{4} \left(\omega \frac{T}{N} \right)^2}_{\exp\left(-\frac{\omega^2 T^2}{4 N^2}\right)^N} \right)^N = \underline{e^{-\frac{\omega^2 T^2}{4N}}} \xrightarrow{N \gg 1} \underline{1}$$

„Quanten-Zeno-Effekt"