letzte Vrlsg.: <u>zeitabhängige Störungstheorie</u>

$$H = H_0 + V(t)$$

H_0: E_n, $|n\rangle$ $\qquad$ $V(t)$: Störung

Trennung von H_0-Dynamik u. $V(t)$-Dynamik

mittels <u>Wechselwirkungsbild</u>:

"Schrödingerzustand" $|\psi(t)\rangle$ genügt

$$i\hbar |\dot{\psi}(t)\rangle = \left(H_0 + V(t)\right) |\psi(t)\rangle$$

WW-bild-Zustand

$$|\psi(t)\rangle_I := e^{iH_0 t/\hbar} |\psi(t)\rangle$$

genügt

$$i\hbar |\dot{\psi}(t)\rangle_I = V_I(t) |\psi(t)\rangle_I \qquad (*)$$

mit

$$V_I(t) := e^{iH_0 t} V(t) e^{-iH_0 t}$$

iterative Lsg. von $(*)$ $\longrightarrow$

$$|\psi(t)\rangle_I = |\psi(0)\rangle - \frac{i}{\hbar}\int_0^t dt' \, \underline{V_I(t')} \, |\psi(0)\rangle$$

$$+ \left(\frac{-i}{\hbar}\right)^2 \int_0^t dt' \int_0^{t'} dt'' \, \underline{V_I(t') \, V_I(t'')} \, |\psi(0)\rangle$$

$$+ \left(\frac{-i}{\hbar}\right)^3 \int_0^t dt' \int_0^{t'} dt'' \int_0^{t''} dt''' \, \underline{V_I(t') \, V_I(t'') \, V_I(t''')} \, |\psi(0)\rangle$$

$$+ \ \ldots$$

<u>Übergangswkt. in 1. Ordnung S.T.</u> :

$|\psi(0)\rangle = |\underline{n}\rangle \xrightarrow[V(t)]{t} |\underline{m}\rangle$ mit Wkt. :

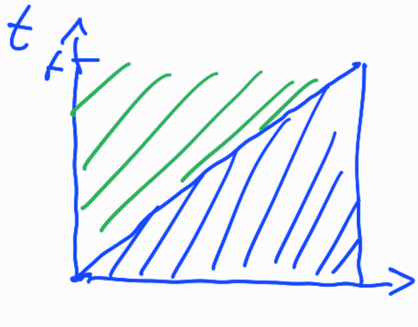
$$\boxed{P_{mn}^{(1)}(t) = \frac{1}{\hbar^2} \left| \int_0^t dt' \, \langle m | V(t) | n \rangle \, e^{i(E_m - E_n)t/\hbar} \right|^2}$$

Zeitentwicklung des Zustands im WW-Bild:

$$|\psi(t)\rangle_I = \Big(\underbrace{\mathbb{1} - \frac{i}{\hbar}\int_0^t dt'\, V_I(t') + \left(\frac{-i}{\hbar}\right)^2 \int_0^t dt' \int_0^{t'} dt''\, V_I(t')\, V_I(t'') + \left(\frac{-i}{\hbar}\right)^3 \int_0^t dt' \int_0^{t'} dt'' \int_0^{t''} dt'''\, V_I(t')\, V_I(t'')\, V_I(t''') + \dots}_{\tilde{U}(t)} \Big)\, |\psi(0)\rangle_I$$

$$= \tilde{U}(t)\, |\psi(0)\rangle_I$$

$$\int_0^t dt' \int_0^{t'} dt''\, V_I(t')\, V_I(t'') = T\, \frac{1}{2} \int_0^t dt' \int_0^t dt''\, V_I(t')\, V_I(t'')$$



$$T\, V(t')\, V(t'') := \begin{cases} V(t')\, V(t'') & : \; t' \geq t'' \\ V(t'') \cdot V(t') & : \; t' < t'' \end{cases}$$

Zeitordnungsoperator!

$$\int^{t} dt' \int^{t'} dt'' \int^{t''} dt''' \, \underline{V_I(t')\, V_I(t'')\, V_I(t''')}$$

$$= \frac{1}{3!}\, \mathcal{T} \int^{t} dt' \int^{t} dt'' \int^{t} dt''' \, \underline{V_I(t')\, V_I(t'')\, V_I(t'''}$$

n-te Ordnung:

$$\left(-\frac{i}{\hbar}\right)^{n} \frac{1}{n!}\, \underline{\underline{\mathcal{T}}} \underbrace{\int_0^{t} dt^{(1)} \int_0^{t} dt^{(2)} \cdots \int_0^{t} dt^{(n)}\, V_I(t^{(1)}) \cdots V_I(t^{(n)})}_{\left(\int_0^{t} dt'\, V_I(t')\right)^{n}}$$

$$= \mathcal{T}\left[\frac{1}{n!}\left(-\frac{i}{\hbar}\int_0^{t} dt'\, V_I(t')\right)^{n}\right].$$

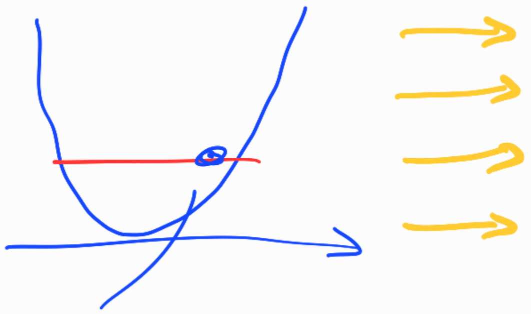
$$\rightarrow \quad \tilde{U}(t) = \underline{\underline{\mathcal{T}}} \exp\left(-\frac{i}{\hbar}\int_0^{t} dt'\, V_I(t')\right)$$

$$\Big[\quad U_0(t) = e^{-iH_0 t/\hbar} = \exp\left(-\frac{i}{\hbar}\int_0^{t} H_0\, dt\right)$$

Harmonische Störung

$$V(t) = u\, e^{i\omega t} + u^+ e^{-i\omega t}$$

[$u = V_0/2 = u^+ \rightarrow V(t) = V_0 \cos(\omega t)$]



m, q

$E_x(t) = E_0 \cos(\omega t)$

$\Rightarrow V(t) = -q E_0 \hat{x} \cos(\omega t)$

H-Atom

m, e

in S.T. 1. Ordnung:

$$P_{nm}(t) = \frac{1}{\hbar} \left| \int_0^t dt' \langle m | V(t') | n \rangle e^{\frac{i(E_m - E_n)t'}{\hbar}} \right|^2$$

$$u\, e^{i\omega t} + u^+ e^{-i\omega t}$$

[$|n\rangle \xrightarrow[V(t)]{?} |m\rangle$

t]

$$\langle m | V(t) | n \rangle = \langle m | a | n \rangle e^{i\omega t} + \langle m | a^{+} | n \rangle e^{-i\omega t}$$

$$= \underset{(a)}{a_{mn}\, e^{i\omega t}} + \underset{(b)}{a_{nm}^{*}\, e^{-i\omega t}}$$

$$P_{nm}(t) = \frac{1}{\hbar^2} \left| \int (a) + \int (b) \right|^2$$

$$\rightarrow P_{nm}^{(b)}(t) = \left| \frac{a_{nm}}{\hbar} \right|^2 \left| \int_0^t dt' \, e^{i \underbrace{\left(\frac{E_m - E_n}{\hbar} - \omega \right)}_{\Omega} t'} \right|^2$$

$$\underline{\underline{NR}}: \left| \int_0^t dt' \, e^{i\Omega t'} \right|^2 = \left| \frac{e^{i\Omega t} - 1}{i\Omega} \right|^2$$

$$= \frac{1}{\Omega^2} \left(2 - 2\cos\Omega t \right) = \frac{2}{\Omega^2} \left(1 - \cos\Omega t \right)$$

$$\cos\Omega t = \cos\left(\frac{\Omega t}{2} + \frac{\Omega t}{2} \right) = \cos^2\frac{\Omega t}{2} - \sin^2\frac{\Omega t}{2}$$

$$= \frac{4}{\Omega^2} \sin^2\frac{\Omega t}{2} = \frac{\sin^2 \Omega/2\, t}{(\Omega/2)^2} ;$$

$$\rightarrow P_{nm}^{(b)}(t) = \left| \frac{a_{nm}}{\hbar} \right|^2 \frac{\sin^2(\Omega/2\, t)}{(\Omega/2)^2} .$$

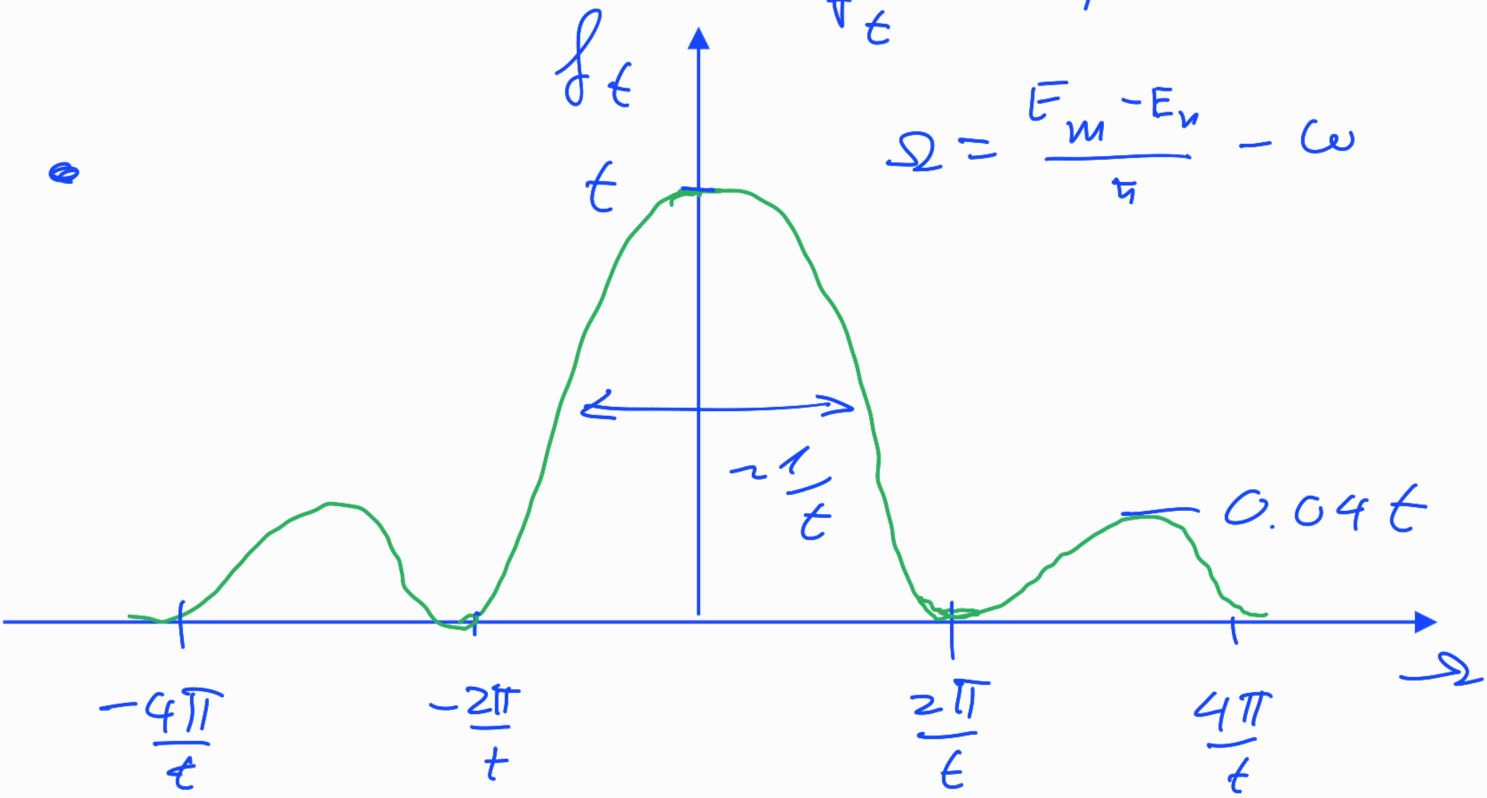
betrachte <u>Übergangsrate</u>:

$$\Gamma_{nm} := \lim_{t\to\infty} \frac{P_{nm}(t)}{t}$$

d.h.

$$\Gamma_{nm}^{(1)} = \left|\frac{u_{nm}}{\hbar}\right|^2 \lim_{t\to\infty} \underbrace{\frac{\sin^2\left(\frac{\Omega}{2}t\right)}{t\left(\frac{\Omega}{2}\right)^2}}_{f_t(\Omega)}\,,$$

$$\Omega = \frac{E_m - E_n}{\hbar} - \omega$$

• 

• $\displaystyle\int_{-\infty}^{\infty} f_t(\Omega) = 2\pi$!

d.h.: $$f_t(\Omega) = \delta_{1/t}(\Omega)\cdot 2\pi$$

$$\to \quad \lim_{t\to\infty} f_t(\Omega) = 2\pi\,\delta(\Omega)$$

Also erhalten wir:

$$\Gamma^{(e)}_{nm} = \left|\frac{u_{nm}}{\hbar}\right|^2 2\pi\,\delta\!\left(\frac{E_m-E_n}{\hbar}-\omega\right)$$

$$\to \quad \Gamma^{(e)}_{nm} = \frac{2\pi}{\hbar}\,|u_{nm}|^2\,\delta(E_m-E_n-\hbar\omega)$$

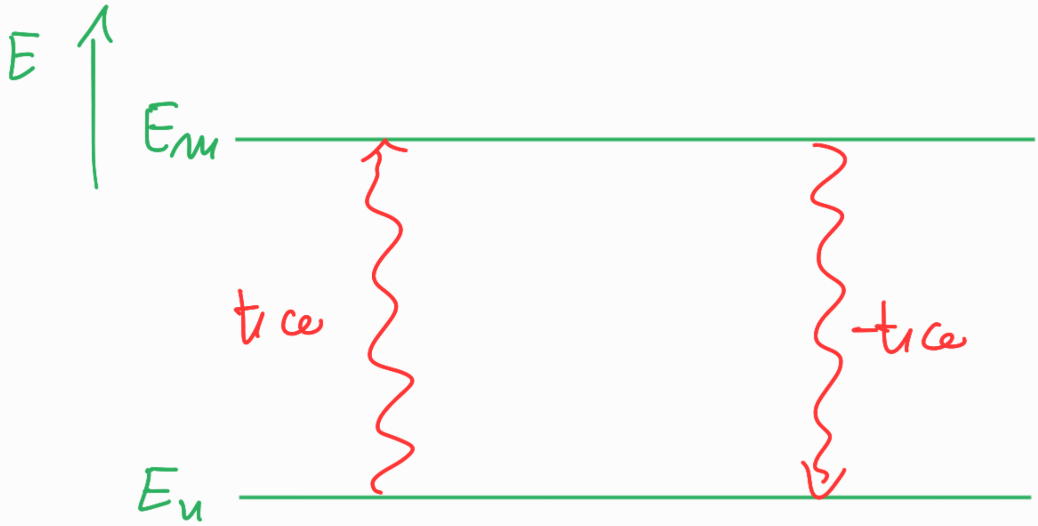
$$\Gamma^{(a)}_{nm} = \frac{2\pi}{\hbar}\,|u_{mn}|^2\,\delta(E_m-E_n+\hbar\omega)$$

Mischterme verschwinden gering $t\to\infty$

$$\boxed{\Gamma_{nm} = \frac{2\pi}{\hbar}\Big(|u_{nm}|^2\,\delta(E_m-E_n-\hbar\omega)^{(e)} + |u_{mn}|^2\,\delta(E_m-E_n+\hbar\omega)^{(a)}\Big)}$$

„Fermis goldene Regel" (Pauli) 1926

physikalische Interpretation:



Absorption eines Photon $\hbar\omega$

induzierte Emission eines Photons $\hbar\omega$

Unbestimmtheitsrelationen

($\Delta p \, \Delta x \geq \hbar/2$

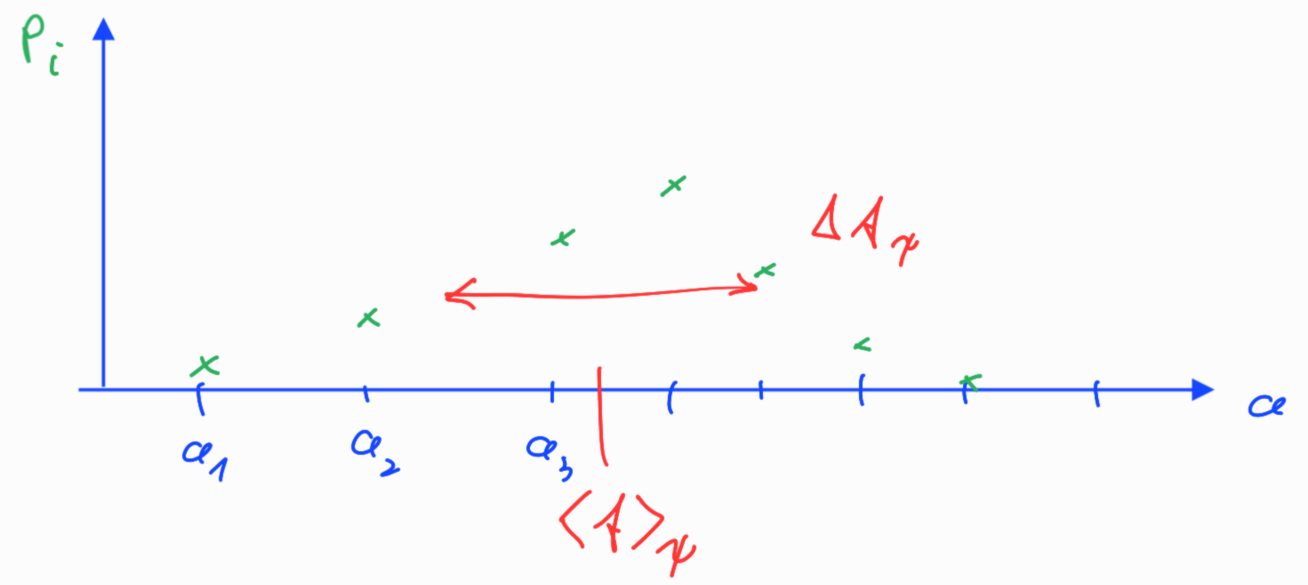
Heisenberg 1927: "Impulsunschärfe Δp aufgrund vorangegangener Ortsmessung mit Unschärfe Δx" ✓)

betrachte Messung von A an System im Zustand $|\psi\rangle$:

Erwartungswert: $\langle A\rangle_\psi = \langle \psi | A | \psi\rangle$

Standardabweichung: $\Delta A_\psi = \left(\langle (A - \langle A\rangle_\psi)^2 \rangle_\psi \right)^{1/2}$

(Varianz: $= (\Delta A_\psi)^2 = \langle (A - \langle A\rangle_\psi)^2 \rangle_\psi$



$\rightsquigarrow$ $$\boxed{\Delta A_\psi \, \Delta B_\psi \geq \frac{1}{2} \left| \langle i [A, B] \rangle_\psi \right|}$$

z.B. $A = \hat{x}$, $B = \hat{p}$ $\rightsquigarrow$ $[x, p] = i\hbar$

$\rightsquigarrow$ $\Delta x_\psi \, \Delta p_\psi \geq \hbar/2$!