Institut für Theoretische Physik
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Theoretische Physik I (Lehramt) – Blatt 13 Lösungen

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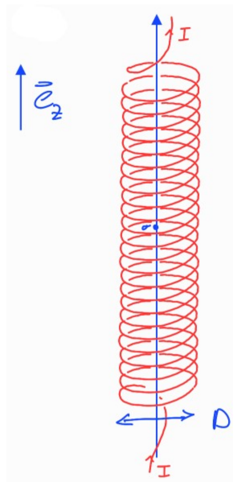
52. Lange Spule

$$\vec{B}(r,\varphi,z) = B_r(r)\vec{e_r} + B_\varphi(r)\vec{e_\varphi} + B_z(r)\vec{e_z} \tag{1}$$

Gauß:



$$0 = \int_{\partial z} \vec{B}\vec{df} = 2\pi r H B_r(r) \tag{2}$$

Integralrechnung:
$\boldsymbol{\partial z}$: Zylinder$\|$ $\vec{e_z}$ **Radius**: r **Höhe**: H **Mittelpunkt**: σ

$$\Rightarrow B_r(r) = 0 \tag{3}$$

$$2\pi r B_\varphi(r) = \int_{\partial s_r} \vec{B}\vec{dl} \tag{4}$$

$$\xrightarrow{Ampre} = \mu_0 I_{s_r} = \begin{cases} 0 & : r < \frac{D}{2} \\ I & : r > \frac{D}{2} \end{cases} \tag{5}$$

Integralrechnung:
$\boldsymbol{\partial s_r}$: kreisscheibe $\perp$ $\vec{e_z}$ **Radius**: r **Mittelpunkt**: σ
Ausßen ($r > \frac{D}{2}$):

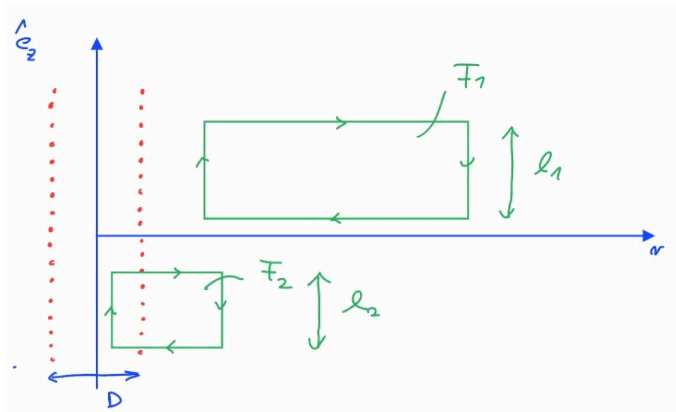
$$B_\varphi(r) = \frac{\mu_0 I}{2\pi r} \tag{6}$$

Innen ($r < \frac{D}{2}$):

$$B_\varphi(r) = 0 \tag{7}$$

nach Ampèresches Gesetz:

$$0 = \int_{\partial F_1} \vec{B}\vec{dl} = l_1(B_Z(N_1) - B_Z(N_2)) \tag{8}$$



Bitte beachten Sie, dass für $r > \frac{D}{2}, \quad B_z(r) = 0$

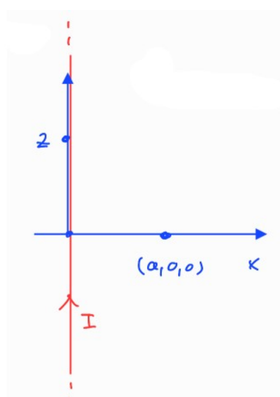
$$\mu_0 l_2 \frac{N}{L} I = \int_{\partial F_2} \overrightarrow{B}\,\overrightarrow{dl} = l_2(B_z(r_i) - B_z(r_a)) = l_2 B_z(r_i) \tag{9}$$

für $r < \frac{D}{2}$: $B_z(r) = \mu_0 N \frac{I}{L}$
d.h.

$$\overrightarrow{B}(r) = \mu_0 I \begin{cases} \frac{1}{2\pi r}\overrightarrow{e_\varphi} & : r > \frac{D}{2} \\ \frac{N}{L}\overrightarrow{e_z} & : r < \frac{D}{2} \end{cases} \tag{10}$$

$$\frac{\left|\overrightarrow{B_{aussen}}\right|}{\left|\overrightarrow{B_{innen}}\right|} \leq \frac{L}{\pi D N} \tag{11}$$

53. Biot-Savart
a)



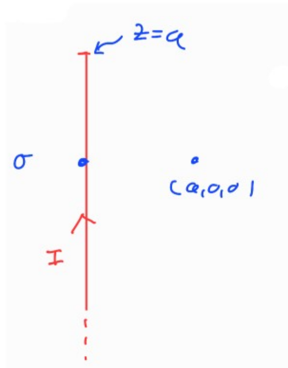
$$z = at \tag{12}$$

$$\overrightarrow{B}(a,0,0) = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty} \frac{\overrightarrow{e_z} \times (a\overrightarrow{e_x} - z\overrightarrow{e_z})}{(z^2+a^2)^{\frac{3}{2}}} dz = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty} \frac{a\overrightarrow{e_y}}{(z^2+a^2)^{\frac{3}{2}}} dz \tag{13}$$

$$= \frac{\mu_0 I \overrightarrow{e_y}}{4\pi}\frac{1}{a}\int_{-\infty}^{\infty} \frac{dt}{(t^2+1)^{\frac{3}{2}}} = \frac{\mu_0 I \overrightarrow{e_y}}{4\pi}\frac{1}{a}\lim_{u\to\infty}\frac{2u}{\sqrt{1+u^2}} \tag{14}$$

$$= \frac{\mu_0 I}{2a\pi}\overrightarrow{e_y} \tag{15}$$

b)



$$\overrightarrow{B_1} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{a} \frac{a\overrightarrow{e_y}}{(z^2+a^2)^{\frac{3}{2}}}dz = \frac{\mu_0 I\overrightarrow{e_y}}{4\pi}\frac{1}{a}\int_{-\infty}^{1}\frac{dt}{(t^2+1)^{\frac{3}{2}}} \tag{16}$$

$$= \frac{\mu_0 I\overrightarrow{e_y}}{4\pi}\frac{1}{a}\lim_{u\to\infty}(\frac{1}{\sqrt{2}} + \frac{u}{\sqrt{1+u^2}}) = \frac{\mu_0 I\overrightarrow{e_y}}{4\pi}\frac{1}{a}(\frac{1}{\sqrt{2}}+1) \tag{17}$$

$$= \frac{\mu_0 I}{4a\pi}(\frac{1}{\sqrt{2}}+1)\overrightarrow{e_y} \tag{18}$$

gleicher Beitrag für $\overrightarrow{B_2}(a,0,0)$

$$\Rightarrow \overrightarrow{B}(a,0,0) = \frac{\mu_0 I}{2a\pi} \tag{19}$$