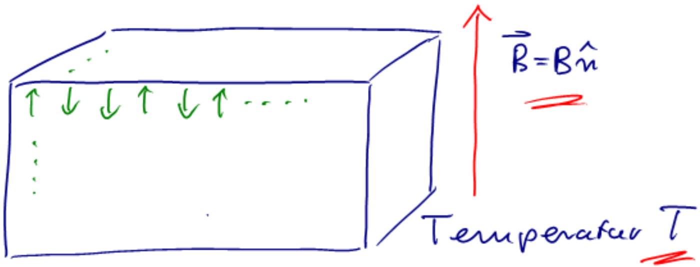
Beispiel 1: Paramagnet im Magnetfeld $\vec{B}$, Temperatur T



$\Rightarrow$ Magnetisierung $m(B,T) = ?$

Gesamtmagnetisierung: $M = \mu_0 \sum_{\ell=1}^{N} s_\ell$

$\rightarrow$ (dimensionslose) Magnetisierung per Spin: $m \equiv \dfrac{M}{\mu_0 N} = \dfrac{1}{N} \sum_{\ell=1}^{N} s_\ell \in [-1,1]$

qualitativ:

(i) B groß, T klein: $\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\cdots$ $\rightarrow$ $m \approx -1$

(ii) B klein, T groß: $\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\cdots$ $\rightarrow$ $m \approx 0$

empirisches Curie-Gesetz: $\boxed{m(B,T) = -c\,\dfrac{B}{T}}$, gültig sofern $|m| \ll 1$

c: materialspezifische Konstante

- warum?
- $c = ?$
- was geschieht wenn $|m| \approx 1$? $\rightarrow$ Stat. Physik! $\rightarrow$

Paramagnet gemäß mikrokanonischer Verteilung:

- mikrokanonische Zustandssumme (aus Vrlsg. 26):

$$Z(E) = \exp\left(N h\left(\frac{1}{2} + \frac{E}{2N\mu_0 B} \right)\right) \qquad (1)$$

$$\hookrightarrow h(x) = -x\ln x - (1-x)\ln(1-x)$$

$$\hookrightarrow h'(x) = \ln\left(\frac{1}{x} - 1\right)$$

$\rightarrow$ mikr. Entropie: $S(E) = k \ln Z(E) \overset{(1)}{=} N k\, h\left(\frac{1}{2} + \frac{E}{2N\mu_0 B}\right)$

$\rightarrow$ Temperatur: $\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{\cancel{N} k}{2\cancel{N}\mu_0 B} h'\left(\frac{1}{2} + \frac{E}{2N\mu_0 B}\right)$

ersetze Energie/Spin durch Magn./Spin: $\frac{E}{N} = \mu_0 B \underbrace{\frac{1}{N}\sum_{l=1}^{N} s_l}_{= m} = \mu_0 B m$

$$\rightsquigarrow \boxed{\frac{2\mu_0 B}{kT} = h'\left(\frac{1}{2} + \frac{m}{2}\right) = \ln\left(\frac{2}{1+m} - 1\right) = \ln\left(\frac{1-m}{1+m}\right)}$$

$h'(x) = \ln\left(\frac{1}{x} - 1\right)$

$\rightarrow$

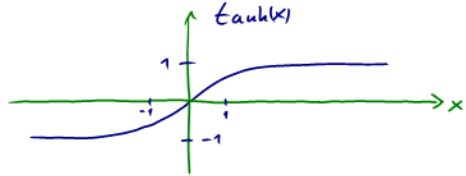
$$\rightarrow \quad \frac{1-m}{1+m} = e^{\frac{2\mu_0 B}{kT}} =: q \qquad \rightarrow \quad 1 - m = q + mq$$

$$\rightarrow \quad \underline{\underline{m}} = \frac{1-q}{1+q} = \frac{q^{-1/2} - q^{1/2}}{q^{-1/2} + q^{1/2}} = \frac{e^{-\mu_0 B/kT} - e^{+\mu_0 B/kT}}{e^{-\mu_0 B/kT} + e^{\mu_0 B/kT}}$$

$$= - \frac{\sinh\left(\frac{\mu_0 B}{kT}\right)}{\cosh\left(\frac{\mu_0 B}{kT}\right)} = \underline{\underline{- \tanh\left(\frac{\mu_0 B}{kT}\right)}}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

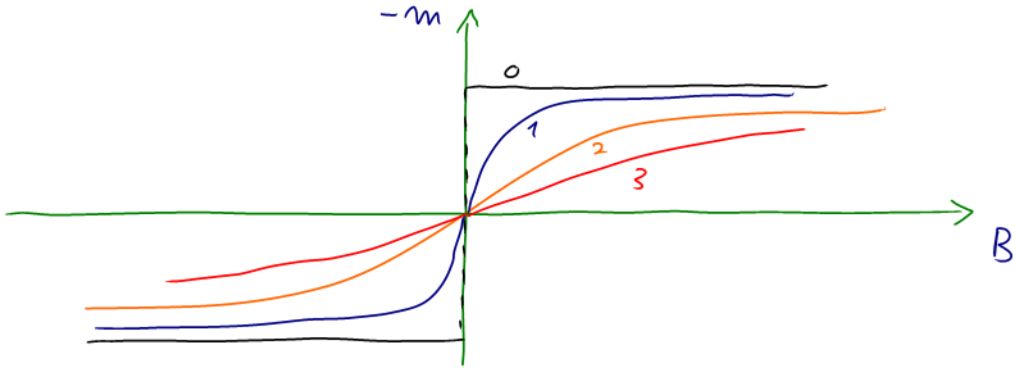
$$\boxed{m(B,T) = - \tanh\left(\frac{\mu_0 B}{kT}\right)}$$



für $\underline{\mu_0 B \ll kT}$ verwende $\tanh x \approx x$: $\quad m(B,T) = - \frac{\mu_0}{k} \cdot \frac{B}{T}$

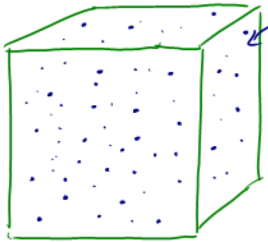
Curie-Gesetz !

$$m(B,T) = -\tanh \frac{\mu_0 B}{kT} :$$



$0 = T_0 < T_1 < T_2 < T_3$

Beispiel 2: Ideales Gas



$N \approx 10^{23}$ Teilchen gleicher Masse m, keine Wechselwirk.

$G \subset \mathbb{R}^3$

$$\rightsquigarrow \quad x = (\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, \vec{p}_1, \vec{p}_2, \dots, \vec{p}_N) \in G^N \times \mathbb{R}^{3N} = \Gamma$$

$$H(x) = \frac{1}{2m} \sum_{i=1}^{N} |\vec{p}_i|^2$$

$$\mathrm{Vol}(G) = \int_G d^3\vec{r} = V$$

$\rightsquigarrow$ mikrokanonische Zustandssumme:

$$Z(E) = \int_{\Omega_{\Delta E}} dx = \int_G d^3\vec{r}_1 \int_G d^3\vec{r}_2 \cdots \int_G d^3\vec{r}_N \cdot \underbrace{\int_{\mathbb{R}^3} d^3\vec{p}_1 \cdots \int_{\mathbb{R}^3} d^3\vec{p}_N}_{E \,\le\, \frac{1}{2m} \sum_i |\vec{p}_i|^2 \,\le\, E + \Delta E} \cdot 1$$

$$Z(E) = \int_{\Omega_{\Delta E}} dx = \underbrace{\underbrace{\int_G d^3\vec{r}_1}_{=V} \underbrace{\int_G d^3\vec{r}_2}_{=V} \cdots \underbrace{\int_G d^3\vec{r}_N}_{=V}}_{V^N} \cdot \underbrace{\int_{\mathbb{R}^3} d^3\vec{p}_1 \cdots \int_{\mathbb{R}^3} d^3\vec{p}_N}_{E \leq \frac{1}{2m}\sum_i |\vec{p}_i|^2 \leq E+\Delta E} \cdot 1$$

$$\underbrace{E \leq \frac{1}{2m}\sum_i |\vec{p}_i|^2 \leq E+\Delta E}$$

$$\Updownarrow$$

$$1 \leq \sum_i \left| \frac{\vec{p}_i}{\sqrt{2mE}} \right|^2 \leq 1 + \frac{\Delta E}{E}$$

$$\frac{\Delta E}{E} =: \gamma \ll 1$$

$$= V^N (2mE)^{\frac{3N}{2}} \cdot \underbrace{\int d^3\vec{u}_1 \cdots \int d^3\vec{u}_N \; 1}_{1 \leq \sum_i |\vec{u}_i|^2 \leq 1+\gamma}$$

<u>Subst.</u>: $\vec{p}_i = (2mE)^{1/2}\,\vec{u}_i$

$$\underbrace{1 \leq \sum_i |\vec{u}_i|^2 \leq 1+\gamma}_{=: \, c}$$

$=: c$, <u>unabhängig</u> von E !

d.h. $$\boxed{Z(E) = \tilde{c}\, V^N E^{3N/2}}$$

→ <u>mikr. Entropie</u>: $S(E) = k \ln Z(E) = kN \ln\left(V E^{3/2}\right) + \tilde{\tilde{c}}$

→ <u>Temperatur</u>: $\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{3}{2} k N \frac{1}{E}$

$$\boxed{\frac{3}{2} kT = \frac{E}{N}} = \frac{1}{N} \sum_{i=1}^{N} \frac{|\vec{p}_i|^2}{2m} = \underbrace{\frac{1}{2m} \overline{|\vec{p}\,|^2}}_{=}$$

<u>kalorische Zustandsgleichung</u> des Idealen Gas'.

mittlere kinetische Energie <u>eines</u> Gasteilchens: $\overline{E}_{kin}^{(1)}$

d.h. $\boxed{\overline{E}_{kin}^{(1)} = \frac{3}{2} kT}$, wie aus elementaren kinetischen Modellen bekannt (vgl. Experimentalphysik 1)