

Lösungshinweise Blatt 3

$$12) \quad |\psi(t)\rangle = e^{-iE_g t/\hbar} |\varphi_g\rangle$$

$$\begin{aligned} \rightarrow \langle A \rangle_t &= \langle \psi(t), A \psi(t) \rangle = \langle e^{-iE_g t/\hbar} \varphi_g, A e^{-iE_g t/\hbar} \varphi_g \rangle \\ &= \langle \varphi_g, A \underbrace{e^{+iE_g t/\hbar} e^{-iE_g t/\hbar}}_{=1} \varphi_g \rangle \\ &= \langle A \rangle_0. \end{aligned}$$

$$13) \quad H = \frac{B\mu_0}{2} (|x-\rangle\langle x-| - |x+\rangle\langle x+|)$$

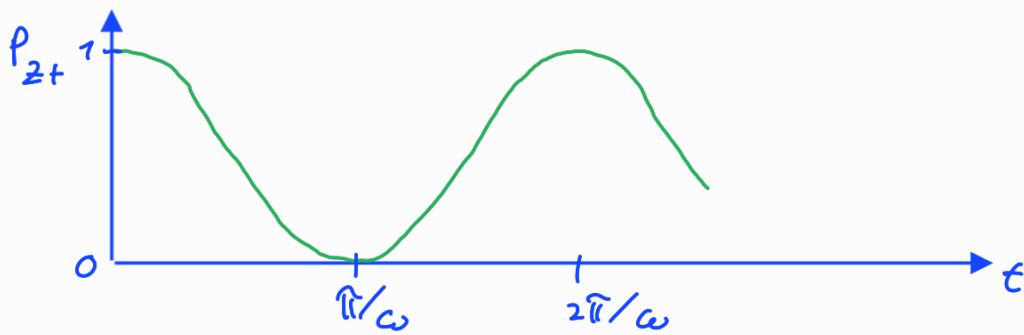
$$\rightarrow e^{-iHt/\hbar} = e^{-i\frac{\omega}{2}t} |x-\rangle\langle x-| + e^{+i\frac{\omega}{2}t} |x+\rangle\langle x+|$$

mit $\omega = B\mu_0/\hbar$ (Larmorfrequenz)

$$\begin{aligned} \rightarrow |\psi(t)\rangle &= e^{-iHt/\hbar} |z+\rangle \\ &= e^{-i\frac{\omega}{2}t} |x-\rangle \underbrace{\langle x-|z+\rangle}_{1/\sqrt{2}} + e^{+i\frac{\omega}{2}t} |x+\rangle \underbrace{\langle x+|z+\rangle}_{1/\sqrt{2}} \end{aligned}$$

$$= \frac{1}{\sqrt{2}} \left(e^{-i\frac{\omega}{2}t} |x-\rangle + e^{+i\frac{\omega}{2}t} |x+\rangle \right)$$

$$\begin{aligned} \rightarrow P_{z+}(t) &= |\langle z+|\psi(t)\rangle|^2 = \frac{1}{2} \left| \frac{e^{-i\frac{\omega}{2}t}}{\sqrt{2}} + \frac{e^{+i\frac{\omega}{2}t}}{\sqrt{2}} \right|^2 \\ &= \frac{1}{4} |1 + e^{i\omega t}|^2 = \frac{1}{4} (2 + 2\cos\omega t) \\ &= \frac{1}{2} (1 + \cos\omega t) \end{aligned}$$



$$\begin{aligned} \mu_2(t) &= \frac{\mu_0}{2} P_{2+} - \frac{\mu_0}{2} P_{2-} = \frac{\mu_0}{2} P_{2+} - \frac{\mu_0}{2} (1 - P_{2+}) \\ &= \mu_0 \left(P_{2+} - \frac{1}{2} \right) = \frac{\mu_0}{2} \cos \omega t \end{aligned}$$

$$\begin{aligned} \mu_x(t) : \quad [H, \mu_x] &= [-B \mu_x, \mu_x] \\ &= -B [\mu_x, \mu_x] = 0 \end{aligned}$$

$\rightarrow \mu_x$ Erhaltungsgröße denn

$$\text{somit } \mu_x(t) = \mu_x(0) = \langle 2+ | \mu_x | 2+ \rangle = 0.$$

$$14) \quad (i) \quad [A+B, C] = (A+B)C - C(A+B)$$

$$= AC + BC - CA - CB = \underbrace{AC - CA}_{[A, C]} + \underbrace{BC - CB}_{[B, C]}$$

$$(ii) \quad [\lambda A, B] = \lambda \underbrace{AB}_{\text{green}} - \underbrace{B\lambda A}_{\text{red}} = \lambda [A, B] = [A, \lambda B]$$

$$\begin{aligned} (iii) \quad A[B, C] + [A, C]B &= (AB)C - \cancel{A}CB + \\ &+ \cancel{A}CB - C(\cancel{A}B) = [AB, C] \end{aligned}$$

$$(iv) \quad [A, BC - CB] + [C, AB - BA] + [B, CA - AC]$$

$$= \cancel{ABC} - \cancel{ACB} - \cancel{BCA} + \cancel{CBA}$$

$$+ \cancel{CAB} - \cancel{CBA} - \cancel{ABC} + \cancel{BAC}$$

$$+ \cancel{BCA} - \cancel{BAC} - \cancel{CAB} + \cancel{ACB}$$

$$= 0.$$