Lösungshinweise Blatt 3

12) $|\psi(t)\rangle = e^{-iE_\ell t/\hbar} |\varphi_\ell\rangle$

$$\Rightarrow \langle A\rangle_t = \langle \psi(t), A\,\psi(t)\rangle = \langle e^{-iE_\ell t/\hbar}\varphi_\ell, A\, e^{-iE_\ell t/\hbar}\varphi_\ell\rangle$$

$$= \langle \varphi_\ell, A\, \underbrace{e^{+iE_\ell t/\hbar}\; e^{-iE_\ell t/\hbar}}_{=1}\, \varphi_\ell\rangle$$

$$= \langle A\rangle_0 .$$

13) $$H = \frac{B\mu_0}{2}\left(|x-\rangle\langle x-| - |x+\rangle\langle x+|\right)$$

$$\Rightarrow e^{-iHt/\hbar} = e^{-i\frac{\omega}{2}t}|x-\rangle\langle x-| + e^{+i\frac{\omega}{2}t}|x+\rangle\langle x+|$$

mit $\omega = B\mu_0/\hbar$ (Larmorfrequenz)

$$\Rightarrow |\psi(t)\rangle = e^{-iHt/\hbar}|z+\rangle$$

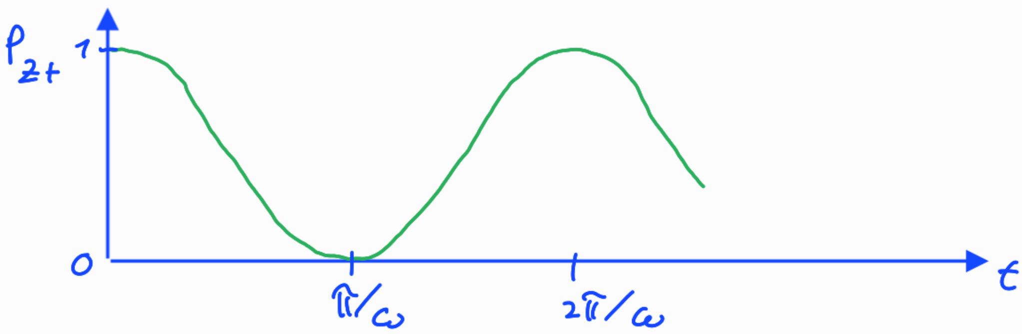
$$= e^{-i\frac{\omega}{2}t}|x-\rangle\underbrace{\langle x-|z+\rangle}_{1/\sqrt{2}} + e^{+i\frac{\omega}{2}t}|x+\rangle\underbrace{\langle x+|z+\rangle}_{1/\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\left(e^{-i\frac{\omega}{2}t}|x-\rangle + e^{i\frac{\omega}{2}t}|x+\rangle\right)$$

$$\Rightarrow P_{z+}(t) = |\langle z+|\psi(t)\rangle|^2 = \frac{1}{2}\left|\frac{e^{-i\frac{\omega}{2}t}}{\sqrt{2}} + \frac{e^{+i\frac{\omega}{2}t}}{\sqrt{2}}\right|^2$$

$$= \frac{1}{4}\left|1 + e^{i\omega t}\right|^2 = \frac{1}{4}(2 + 2\cos\omega t)$$

$$= \frac{1}{2}(1 + \cos\omega t)$$



- $\mu_z(t) = \frac{\mu_0}{2} P_{z+} - \frac{\mu_0}{2} P_{z-} = \frac{\mu_0}{2} P_{z+} - \frac{\mu_0}{2}(1 - P_{z+})$

$$= \mu_0 \left(P_{z+} - \frac{1}{2} \right) = \frac{\mu_0}{2} \cos \omega t$$

- $\mu_x(t)$: $\quad [H, \mu_x] = [-B\mu_x, \mu_x]$

$$= -B[\mu_x, \mu_x] = 0$$

$\rightarrow$ μ_x Erhaltungsgröße und

somit $\mu_x(t) = \mu_x(0) = \langle z+|\mu_x|z+\rangle = 0$.

14) (i) $\quad [A + B, C] = (A+B)C - C(A+B)$

$$= AC + BC - CA - CB = \underbrace{AC - CA}_{[A,C]} + \underbrace{BC - CB}_{[B,C]}$$

(ii) $\quad [\lambda A, B] = \lambda A B - B \lambda A = \lambda [A, B] = [A, \lambda B]$

(iii) $\quad A[B, C] + [A, C]B = (AB)C - \cancel{ACB} +$

$$+ \cancel{ACB} - C(AB) = [AB, C]$$

(iv) $[A, BC - CB] + [C, AB - BA] + [B, CA - AC]$

$$= \cancel{ABC} - \cancel{ACB} - \cancel{BCA} + \cancel{CBA}$$

$$+ \cancel{CAB} - \cancel{CBA} - \cancel{ABC} + \cancel{BAC}$$

$$+ \cancel{BCA} - \cancel{BAC} - \cancel{CAB} + \cancel{ACB}$$

$$= 0 .$$