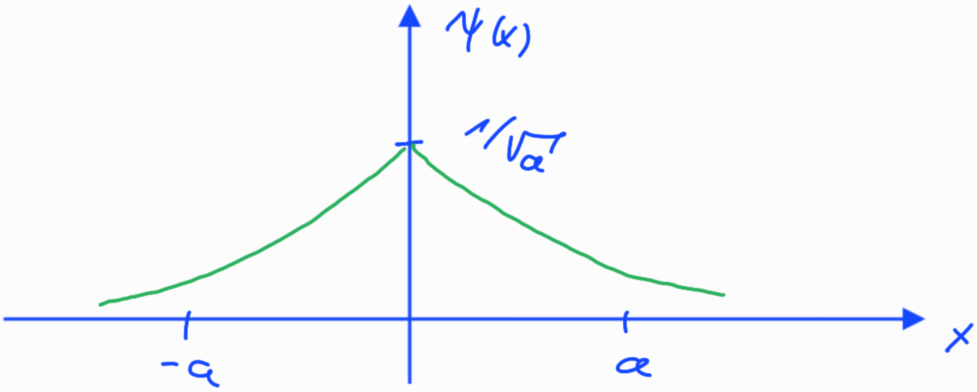
Lösungshinweise Blatt 4

16 a)



b)

$$|\psi| = \langle\psi|\psi\rangle = \int_{-\infty}^{\infty} dx\, |\psi(x)|^2 = \int_{-\infty}^{\infty} dx\, \frac{e^{-2|x|/a}}{a}$$

$$= \frac{2}{a}\int_0^{\infty} dx\, e^{-2x/a} = \frac{2}{a}\left(-\frac{a}{2}\, e^{-\frac{2x}{a}}\right)_0^{\infty} = 1$$

c)

$$\langle x\rangle_\psi = \langle\psi|x|\psi\rangle = \int_{-\infty}^{\infty} dx\, \underbrace{x\,\frac{e^{-2|x|/a}}{a}}_{\text{ungerade}} = 0$$

$$\langle x^2\rangle_\psi = \langle\psi|x^2|\psi\rangle = \int_{-\infty}^{\infty} dx\, x^2\,\frac{e^{-2|x|/a}}{a}$$

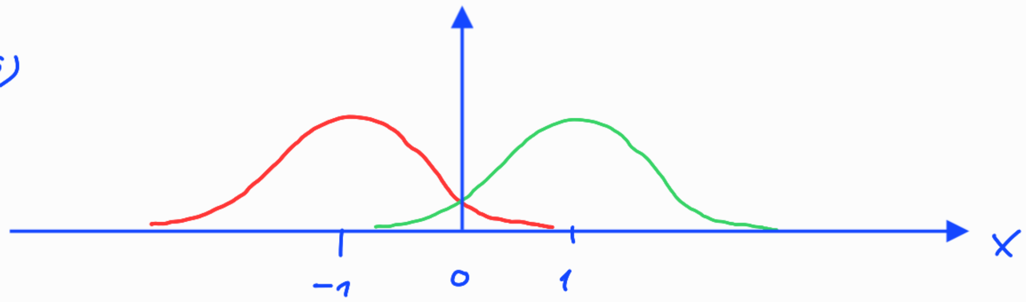
$$= \frac{2}{a}\int_0^{\infty} dx\, x^2 e^{-2x/a} \underset{x=\frac{a}{2}y}{=} \frac{a^2}{4}\int_0^{\infty} dx\, y^2 e^{-y} \overset{(i)}{=} \frac{a^2}{2}$$

d)

$$P(x>a) = \int_a^{\infty} dx\, |\psi(x)|^2 = \frac{1}{a}\int_a^{\infty} e^{-2x/a}$$

$$= \frac{1}{a}\left(-\frac{a}{2}\, e^{-2x/a}\right)_a^{\infty} = \frac{1}{2e^2} = 0.068\ldots$$

17 a)



b) $\psi_{1/2}(x)$ symmetrisch um $x_{1/2} = \pm a$

$\rightarrow \langle x \rangle_{\psi_1} = a \,, \quad \langle x \rangle_{\psi_2} = -a$

c)

- $\langle \psi_1 | \psi_2 \rangle = \underbrace{\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} dx \; e^{-\frac{x^2}{2\sigma^2}}}_{\overset{(ii)}{=} 1} e^{-a^2/2\sigma^2} = e^{-\frac{a^2}{2\sigma^2}}$

- $\langle \psi_1 | \hat{x} | \psi_2 \rangle = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} dx \; \underbrace{x \, e^{-x^2/2\sigma^2}}_{\text{ungerade}} e^{-a^2/2\sigma^2} = 0$

- $\langle \psi_1 | \hat{x}^2 | \psi_2 \rangle = \frac{1}{\sqrt{2\pi\sigma_2}} \int_{-\infty}^{\infty} dx \; x^2 e^{-x^2/2\sigma^2} \; e^{-a^2/2\sigma^2}$

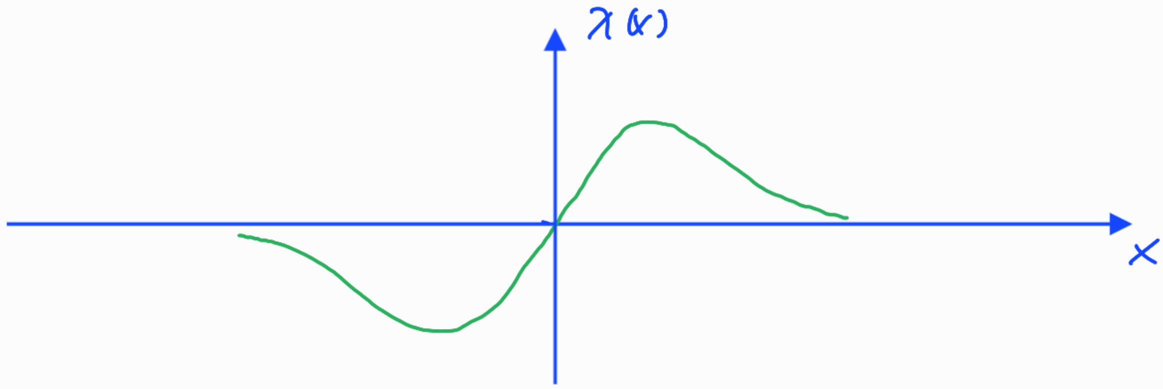
$\underset{x = \sqrt{2\sigma^2}\, y}{=} \frac{2\sigma^2}{\sqrt{\pi}} e^{-a^2/2\sigma^2} \underbrace{\int_{-\infty}^{\infty} dy \; y^2 e^{-y^2}}_{= \frac{\sqrt{\pi}}{2}} = \sigma^2 e^{-\frac{a^2}{2\sigma^2}}$

d) $\quad 1 \overset{!}{=} \langle \chi | \chi \rangle = \alpha^2 \left(\langle \psi_1 | - \langle \psi_2 | \right) \left(|\psi_1\rangle - |\psi_2\rangle \right)$

$$= \alpha^2 \Big(\underbrace{|\psi_1|^2}_{=1} + \underbrace{|\psi_2|^2}_{=1} - \underbrace{\langle \psi_2 | \psi_1 \rangle}_{= e^{-a^2/2\sigma^2}} - \underbrace{\langle \psi_1 | \psi_2 \rangle}_{= e^{-a^2/2\sigma^2}} \Big)$$

$$= 2\alpha^2 \left(1 + e^{-a^2/2\sigma^2} \right)$$

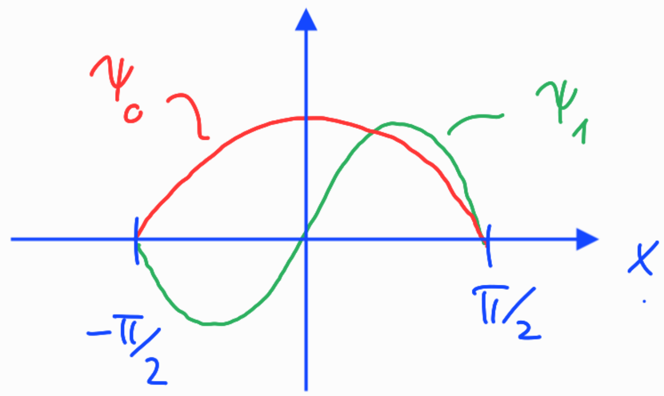
$$\rightarrow \quad \alpha = \left(2 + 2 e^{-a^2/2\sigma^2} \right)^{-1/2}$$



$\langle x \rangle_\chi = 0$

↖ Symmetrie von $|\chi(x)|^2$ um $x=0$

18 a)



b) Symmetrie von $|\psi_{0/1}(x)|^2$ um $x=0$

$\rightarrow \quad \langle \psi_0 | \hat{x} | \psi_0 \rangle = 0 \,, \quad \langle \psi_1 | \hat{x} | \psi_1 \rangle = 0$

$$\bullet \quad \langle \psi_0 | \hat{x} | \psi_1 \rangle = \frac{2}{\pi} \underbrace{\int_{-\pi/2}^{\pi/2} dx \; x \cos x \, \sin 2x}_{\stackrel{(iv)}{=} \frac{8}{9}} = \frac{16}{9\pi}$$

c)

$$|\chi(t)\rangle = \frac{e^{-iE_0 t/\hbar}}{\sqrt{2}} |\psi_0\rangle + \frac{e^{-iE_1 t/\hbar}}{\sqrt{2}} |\psi_1\rangle$$

d)

$$\langle x \rangle_t = \langle \chi(t) | \hat{x} | \chi(t) \rangle$$

$$= \frac{1}{2} \underbrace{\langle x \rangle_{\psi_0}}_{=0} + \frac{1}{2} \underbrace{\langle x \rangle_{\psi_1}}_{=0} + \frac{1}{2} e^{i(E_1 - E_0)t/\hbar} \underbrace{\langle \psi_1 | x | \psi_0 \rangle}_{= 16/9\pi} + \frac{1}{2} e^{-i(E_1 - E_0)t/\hbar} \underbrace{\langle \psi_2 | x | \psi_0 \rangle}_{= 16/9\pi}$$

$$= \frac{16}{9\pi} \cos(\omega t), \qquad \omega = \frac{E_1 - E_0}{\hbar}$$

Teilchen oszilliert mit Freq. ω im Kasten hin und her.