Lösungshinweise Blatt 7

28) nach Vorlsg. $p = e^{-\frac{1}{\hbar}\sqrt{8m(U-E)}\,d}$;

d.h. $0.1 \equiv P_{0.9U} = \Big(\underbrace{e^{-d\sqrt{8mU}/\hbar}}_{=:\, b}\Big)^{\sqrt{0.1}} = b^{\sqrt{0.1}}$

$\rightarrow \quad P_{0.1U} = b^{\sqrt{0.9}} = (P_{0.9U})^{\sqrt{0.9}/\sqrt{0.1}} = P_{0.9U}^{3} = 10^{-3}.$

29)

a) $$\underbrace{\int_{-\varepsilon}^{\varepsilon} E\,\psi(x)\,dx}_{\downarrow\, \varepsilon\to 0 \;\; 0} \overset{!}{=} -\frac{\hbar^2}{2m}\underbrace{\int_{-\varepsilon}^{\varepsilon}\psi''(x)\,dx}_{\downarrow\,\varepsilon\to 0\;\;\Delta\psi'} + \underbrace{\int_{-\varepsilon}^{\varepsilon} u\,\delta(x)\,\psi(x)\,dx}_{= u\,\psi(0)}$$

$$\rightarrow \quad \Delta\psi' = \frac{2mu}{\hbar^2}\,\psi(0) \qquad (1)$$

b) • ψ stetig in $x=0$: $\rightarrow \quad 1 + r = t$

• Relation (1): $\rightarrow \quad t - 1 + r = -i\,\frac{2mu}{k\,\hbar^2}\,t$

$$\rightarrow \quad t - 1 = -\,i\,\frac{mu}{k\hbar^2}\,t$$

$$\rightarrow \quad \bullet\; t = \frac{1}{1 + i\,\frac{mu}{k\hbar^2}}$$

$$\bullet\; r = t - 1 = \frac{1}{i\,\frac{k\hbar^2}{mu} - 1}$$

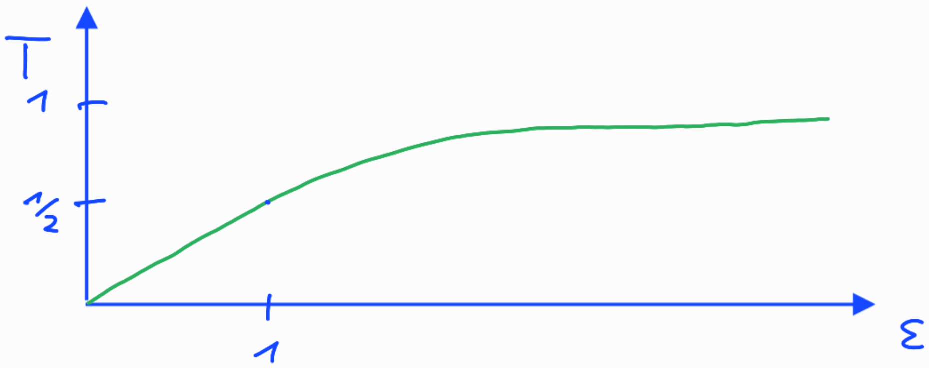
1) $$T = |t|^2 = \left(1 + \left(\frac{mu}{\hbar^2 k}\right)^2\right)^{-1}$$

$$= \left(1 + \frac{mu^2}{2\hbar^2 E}\right)^{-1} = \left(1 + E_0/E\right)^{-1}$$

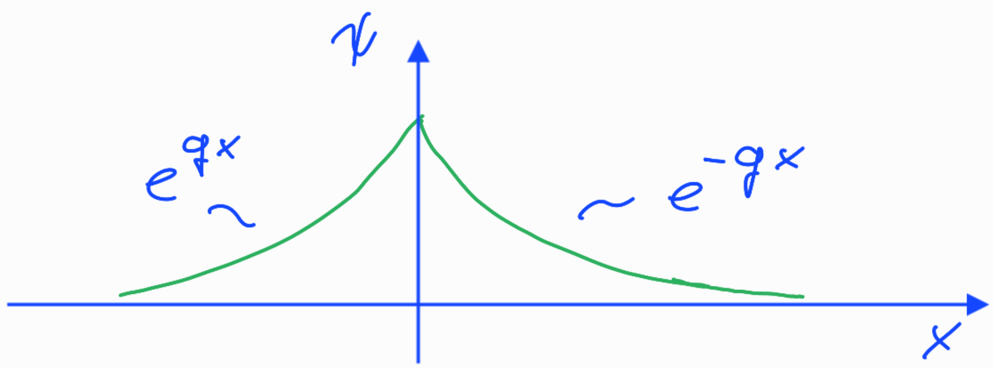
$k = \sqrt{2mE}/\hbar$ $\qquad$ $E_0 = \frac{mu^2}{2\hbar^2}$

$\to T(E_0) = 1/2$ ✓

und $T(\varepsilon) = \frac{1}{1+1/\varepsilon} = \frac{\varepsilon}{\varepsilon+1}$.



30) für $x \neq 0$ ist $\psi_E(x)$ normierbare Lsg. der freien stat. SG. $E\psi = -\frac{\hbar^2}{2m}\psi''$, d.h. $\psi(x) = c \cdot e^{-q|x|}$ mit $q = \sqrt{2m|E|}/\hbar$.



Normierung:

$$1 = \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 2c^2 \int_0^{\infty} e^{-qx} dx = \frac{2c^2}{q}$$

$$\rightarrow \quad c = \sqrt{2q} \quad , \quad \psi(x) = \sqrt{2q}\, e^{-q|x|}$$

$$\rightarrow \quad \Delta\psi' = \lim_{\varepsilon \searrow 0} 2\psi'(\varepsilon) = -\sqrt{2q} \cdot 2q$$

$$\overset{!}{=} -\frac{2m\alpha}{\hbar^2}\psi(0) = -\frac{2m\alpha}{\hbar^2}\sqrt{2q}$$

$$\rightarrow \quad q = \frac{m\alpha}{\hbar^2} \overset{!}{=} \sqrt{2m|E|}/\hbar$$

$$\rightarrow \quad \underline{\underline{E = -\frac{m\alpha^2}{2\hbar^2}}}$$

31)

$$|\psi_1\rangle = |a_1 b_1\rangle + |a_2 b_2\rangle + |a_1 b_2\rangle$$

$$\overset{?}{=} (\alpha_1 |a_1\rangle + \alpha_2 |a_2\rangle) \otimes (\beta_1 |b_1\rangle + \beta_2 |b_2\rangle)$$

$$= \underbrace{\alpha_1 \beta_1}_{1=} |a_1 b_1\rangle \qquad = 0$$

$$+ \underbrace{\alpha_1 \beta_2}_{1=} |a_1 b_2\rangle$$

$$+ \underbrace{\alpha_2 \beta_1}_{0=} |a_2 b_1\rangle \rightarrow \quad \alpha_2 = 0 \quad \vee \quad \beta_1 = 0$$

$$+ \underbrace{\alpha_2 \beta_2}_{1=} |a_2 b_2\rangle \qquad = 0$$

$\rightsquigarrow \quad |\psi_1\rangle \neq |\phi_A\rangle \otimes |\phi_B\rangle$ für alle

$|\phi_A\rangle, |\phi_B\rangle$

d.h. $|\psi_1\rangle$ verschränkt.

Offenbar:

$$|\psi_2\rangle \overset{!}{=} (|a_1\rangle + |a_2\rangle) \otimes (|b_1\rangle + |b_2\rangle)$$

$$|\psi_3\rangle \overset{!}{=} (|a_1\rangle - |a_2\rangle) \otimes (|b_1\rangle - |b_2\rangle)$$

daher $|\psi_2\rangle, |\psi_3\rangle$ separabel.