Letzte Vorlesung:

1. Postulat: (Superpositionsprinzip)

$$\boxed{\begin{array}{l}\text{Zustandsraum} \mathrel{\hat{=}} \text{unitärer}^{(*)} \text{ VR } \mathcal{H} \\ \text{Zustand} \mathrel{\hat{=}} \text{normierter Vektor } \varphi \in \mathcal{H}\end{array}}$$

(*) d.h. $\mathcal{H}$ komplexer VR mit hermit. Skalarprod.

$\varphi \in \mathcal{H}, \lambda \in \mathbb{C} \to \lambda \varphi \in \mathcal{H}$

$\varphi, \psi \longmapsto \langle \varphi, \psi \rangle$:

- $\langle \varphi, \psi \rangle = \langle \psi, \varphi \rangle^*$
- $\langle \varphi, \varphi \rangle \geq 0 \quad (\varphi \neq \vec{0})$
- $\langle \varphi, \lambda \psi \rangle = \lambda \langle \varphi, \psi \rangle$
  $\langle \varphi, \psi + \chi \rangle = \langle \varphi, \psi \rangle + \langle \varphi, \chi \rangle$

Norm: $|\varphi| := \sqrt{\langle \varphi, \varphi \rangle}$

Orthogonalität:

$\varphi \perp \psi \; :\Longleftrightarrow \; \langle \varphi, \psi \rangle = 0$

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2. Postulat (elementare Version)

a) Zust. $\varphi \longrightarrow$ Messung M_φ :

"1", "positiv" ("φ liegt vor")

"0", "negativ" ("φ liegt nicht vor")

b) Messung M_φ an System im Zust. ψ positiv mit Wkt. $p = |\langle \varphi, \psi \rangle|^2$.

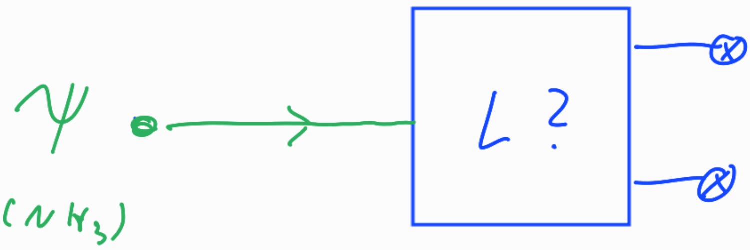
Zusatz: Messung M_φ <u>ideal</u> $:\Longleftrightarrow$ <u>nach</u> positiver Messung System im Zustand φ

Beispiel:

 : $\varphi_L \in \mathcal{H}_D$, $\quad |\varphi_L| = 1$

 : $\varphi_R \in \mathcal{H}_D$, $\quad |\varphi_R| = 1$

Messung M_{φ_L}



"1" : $p = |\langle \varphi_L, \psi \rangle|^2$

"0" : $1 - p$

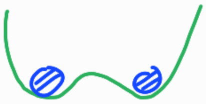
z. B.:

- $\underline{\psi = \varphi_L}$: $\longrightarrow \quad p = |\langle \varphi_L, \varphi_L \rangle|^2 = 1$ ✓

- $\underline{\psi = \varphi_R}$: $\quad 0 \overset{!}{=} p = |\langle \varphi_L, \varphi_R \rangle|^2$

d. h. $\langle \varphi_L, \varphi_R \rangle \overset{!}{=} 0$, also $\boxed{\varphi_L \perp \varphi_R}$

Messung M_{φ_L} an Überlagerung:

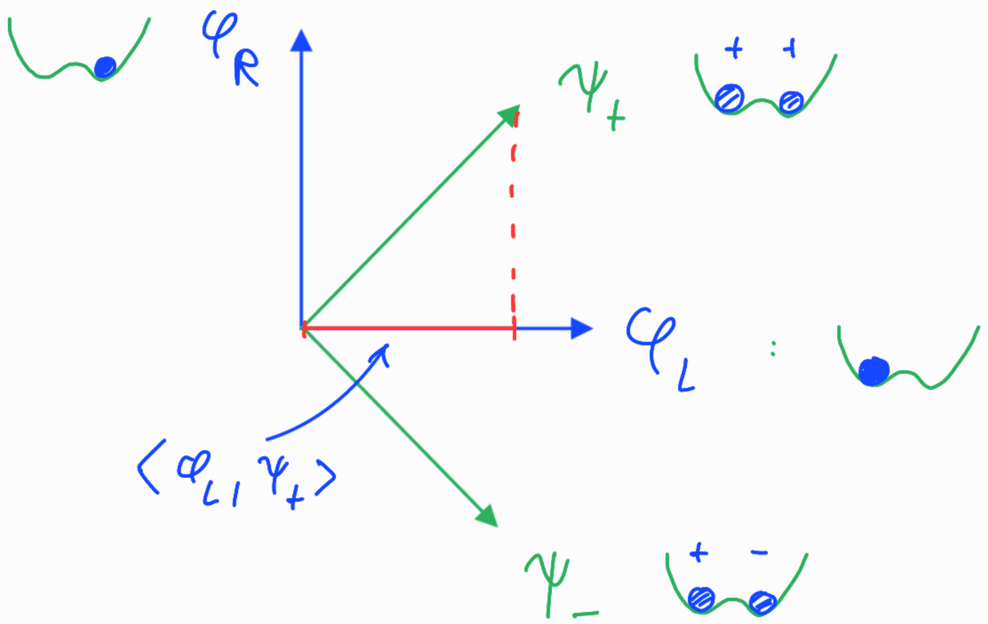


: $\psi_\pm = \frac{1}{\sqrt{2}} \left(\varphi_L \pm \varphi_R \right)$:

$$\rightarrow \quad p = \left| \langle \varphi_L, \psi_+ \rangle \right|^2 = \left| \langle \varphi_L, \tfrac{1}{\sqrt{2}} (\varphi_L \pm \varphi_R) \rangle \right|^2$$

$$= \left| \frac{1}{\sqrt{2}} \underbrace{\langle \varphi_L, \varphi_L \rangle}_{=1} \pm \frac{1}{\sqrt{2}} \underbrace{\langle \varphi_L, \varphi_R \rangle}_{=0} \right|^2$$

$$= 1/2$$

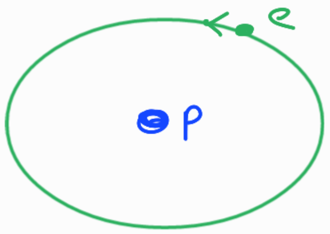


Stern-Gerlach-Versuch

(1922, Frankfurt)

Hintergrund: <u>Drehimpulsquantisierung</u>

<u>Bohrsche Atommodell (1913)</u>:



klassische Bahn des Elek. um Proton (H-Atom)

mit quantisierten Drehimpuls:

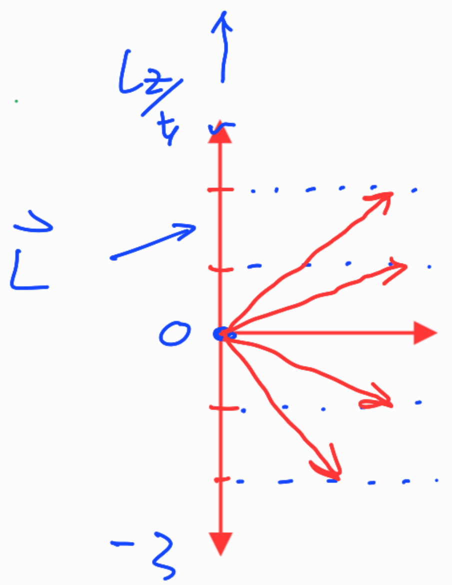
$$\boxed{|\vec{L}| \overset{!}{=} \ell \hbar \qquad \ell = 1, 2, 3, \dots}$$

Sommerfeld (1916):

$$\hbar = \frac{h}{2\pi} = 1.0545\ldots 10^{-34}\,\mathrm{Js}$$

$$L_z \overset{!}{=} m \hbar$$

$$m = -\ell, -\ell+1, \dots, \ell-1, \ell$$



"Richtungs-quantisierung"

im isotropen Raum? "Unsinn!"

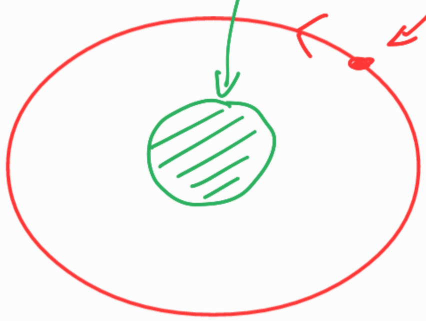
$[\hbar] = \text{Energie} \cdot \text{Zeit} = \text{Wirkung}$

$= \text{Länge} \cdot \text{Impuls}$

$= \text{Drehimpuls}$

Experiment mit Silberatomen:

Ag-Atom: $\underbrace{[Kr]\,4d^{10}}\,5s^1$

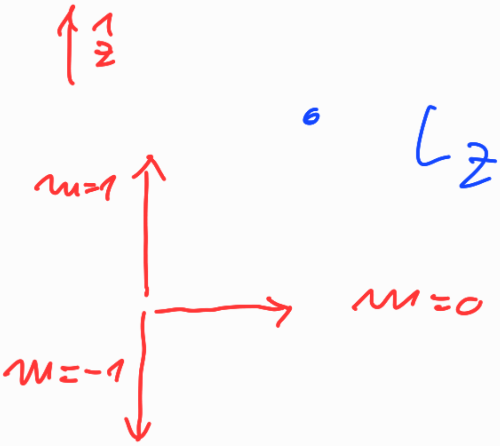


Drehimpuls $\vec{L}$ des $5s^1$ Elektrons

quantisiert: • $|\vec{L}| = l\,\hbar$ $\quad l = 1$

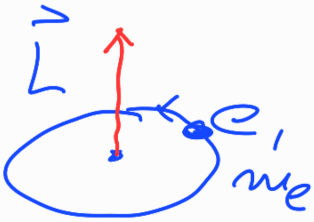
(falsche Vorstellung, da Elektron-Spin noch nicht bekannt)

• $L_z = m\,\hbar \qquad m = -1, 0, +1$



Messung von L_z ??

Drehimpuls $\longleftrightarrow$ mag. Moment



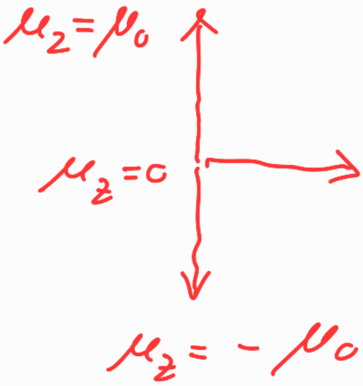
$$\boxed{\vec\mu \overset{!}{=} \frac{e}{2m_e}\vec L}$$

- $|\vec L| = l\hbar \quad \longleftrightarrow \quad |\vec\mu| = \mu_0 \cdot l$

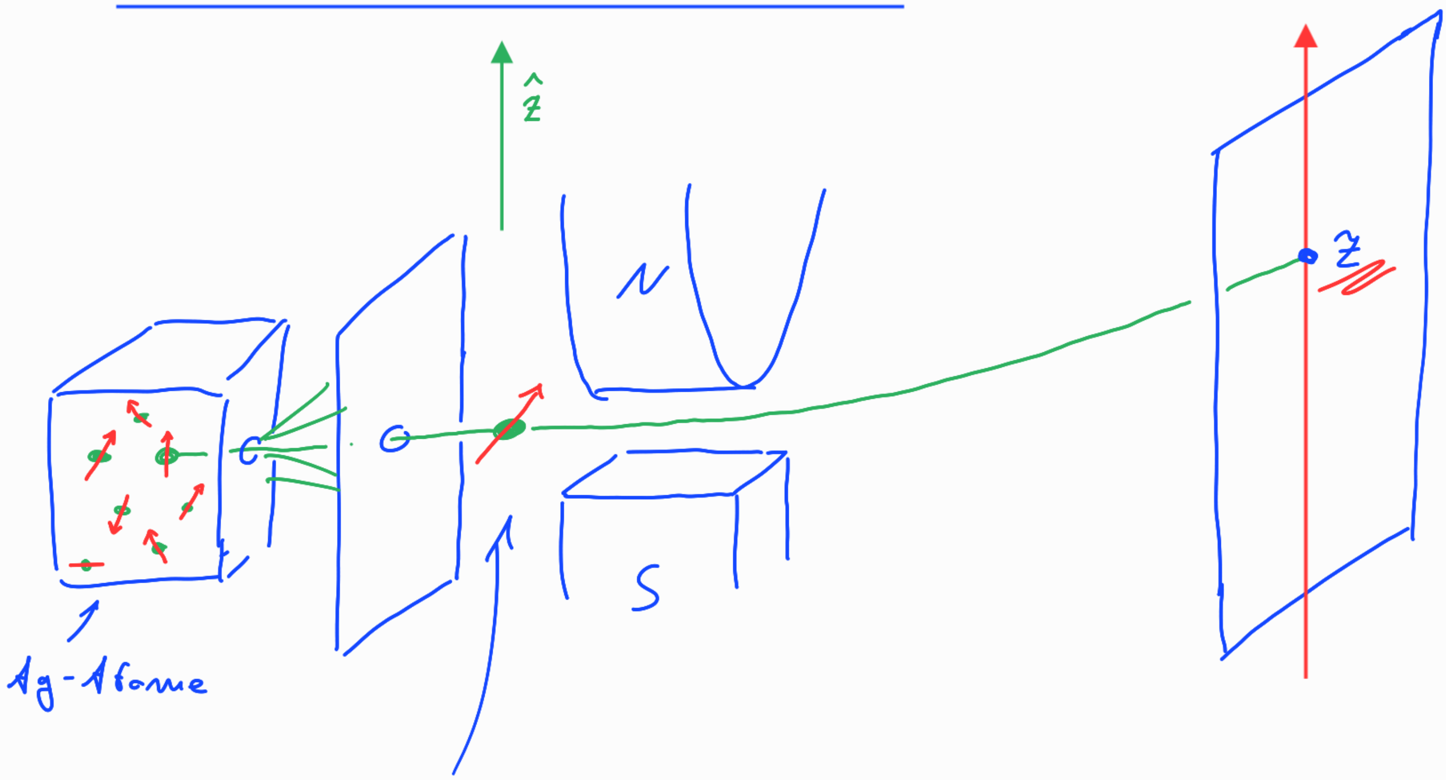
Bohrsches Magneton:
$\mu_0 = e\hbar/2m_e$

- $L_z = m\hbar \quad \longleftrightarrow \quad \mu_z = \mu_0\, m$

Quantisierung des mag. Moments:



→ Stern-Gerlach-Versuch:



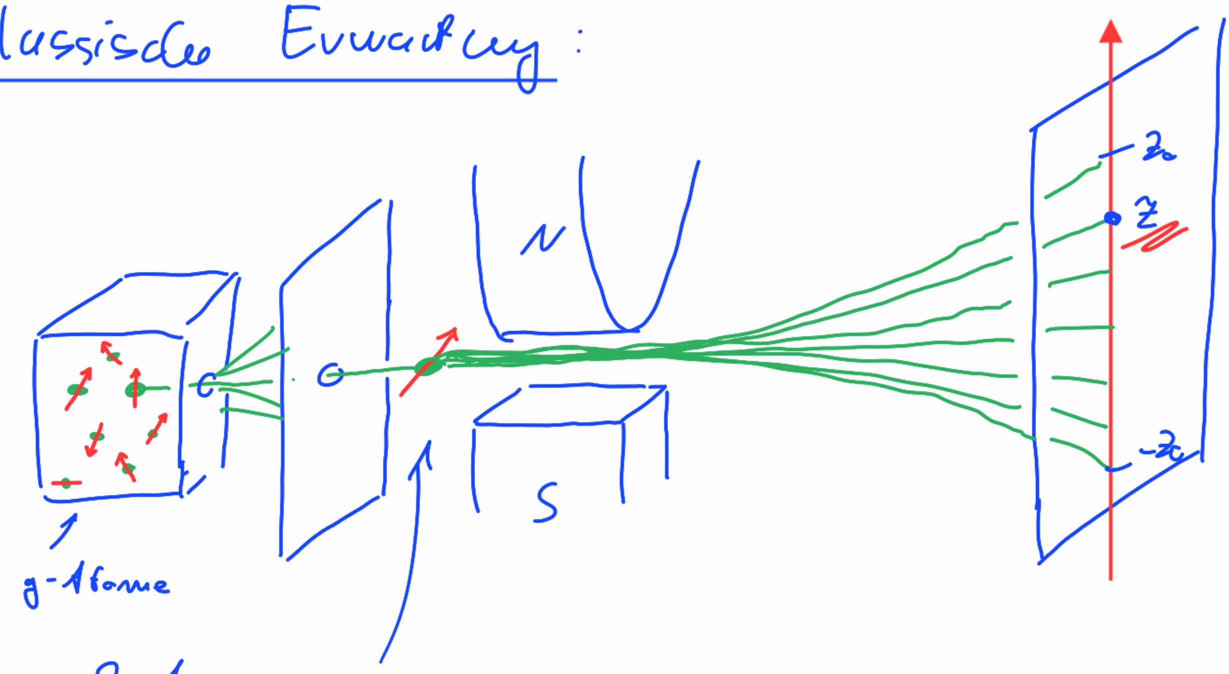
Ag-Atom erfährt Kraft im inhomogenen Mag.feld:

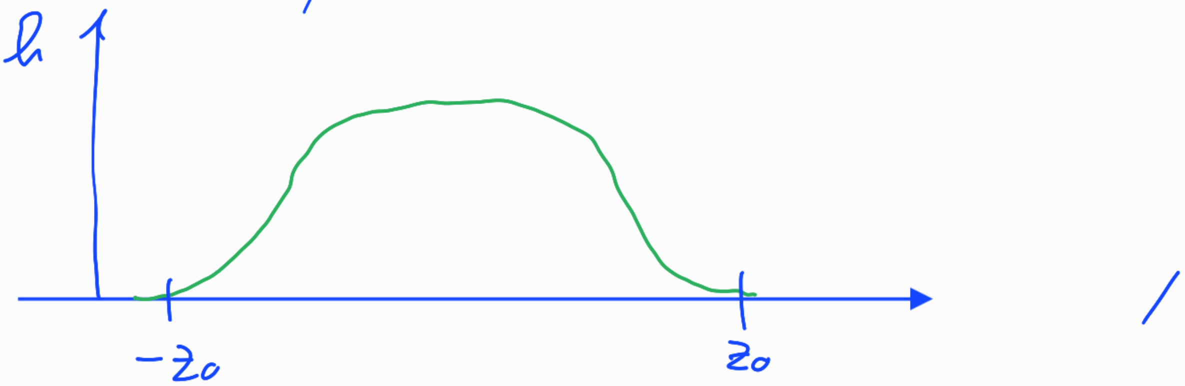
$$\vec{F} = \frac{\partial B_z}{\partial z} \cdot \mu_z \cdot \hat{z}$$

$$\boxed{z \propto \mu_z}$$

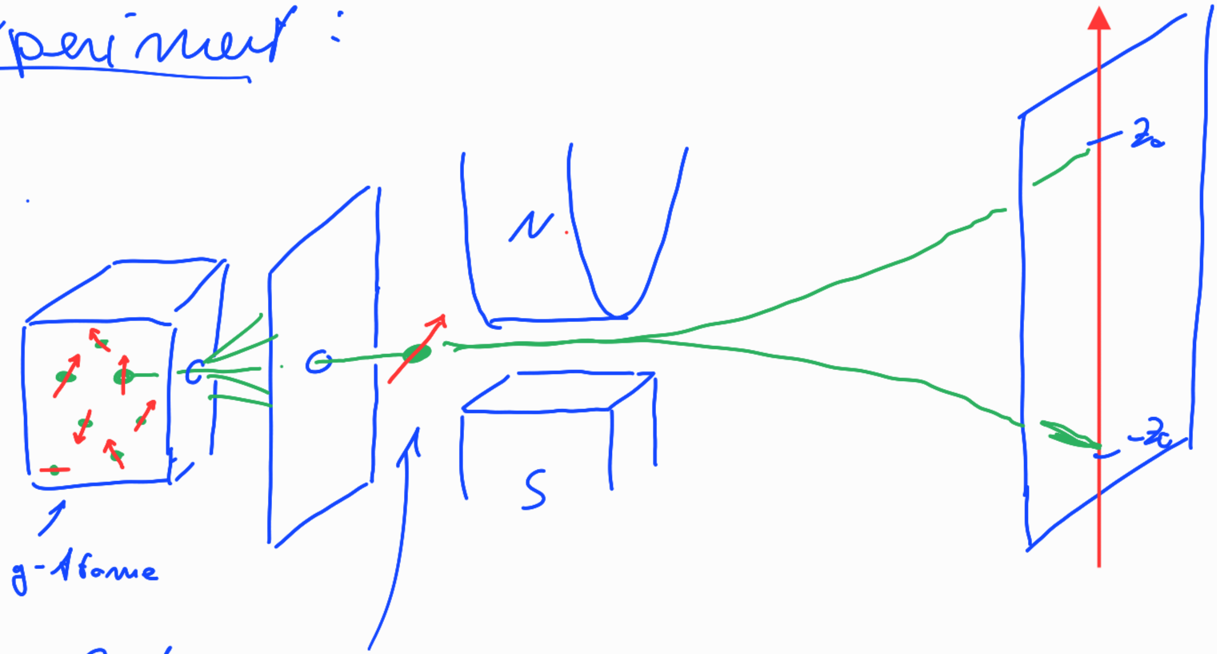
QM : kl Ph = 1 : 0 !

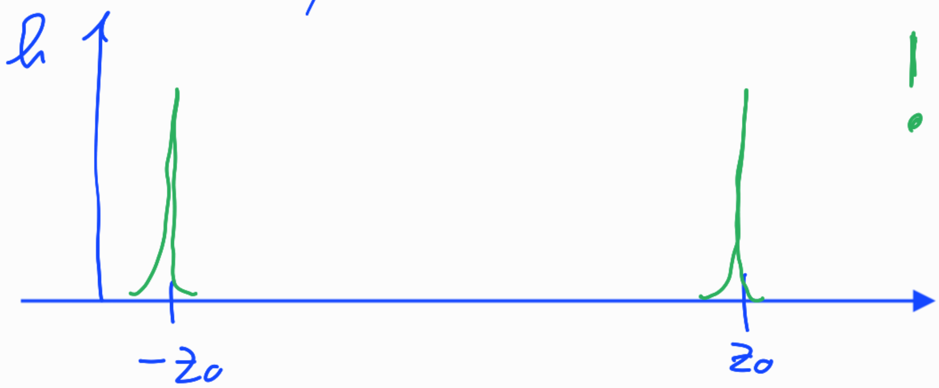
klassische Erwartung:



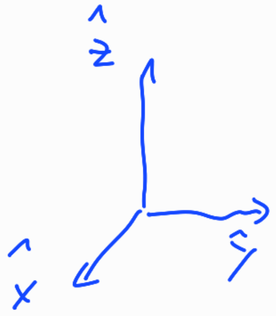


Experiment:

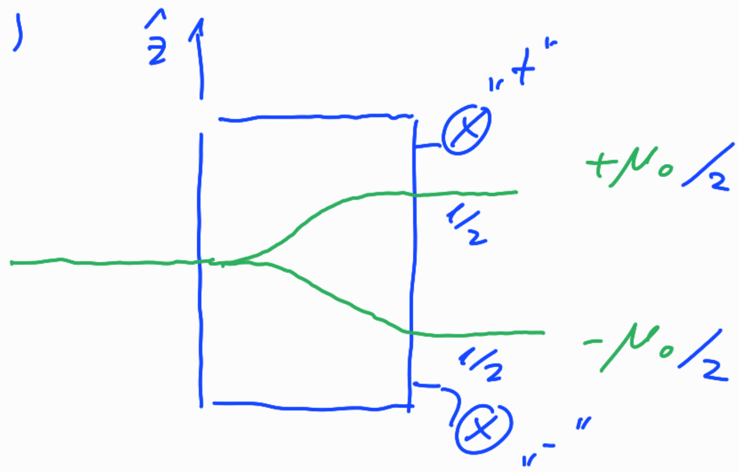




schematisch:

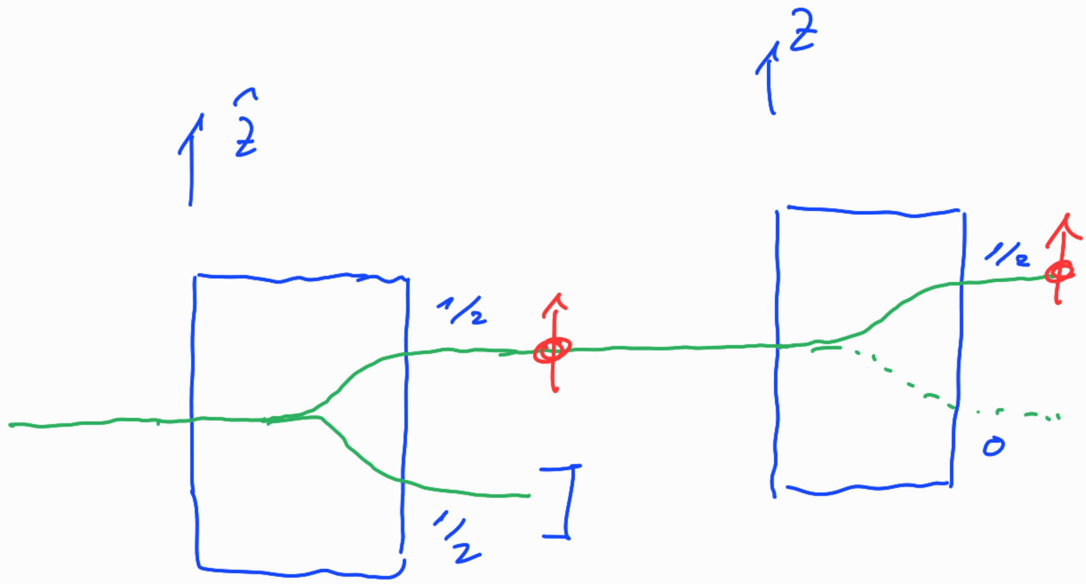


a)

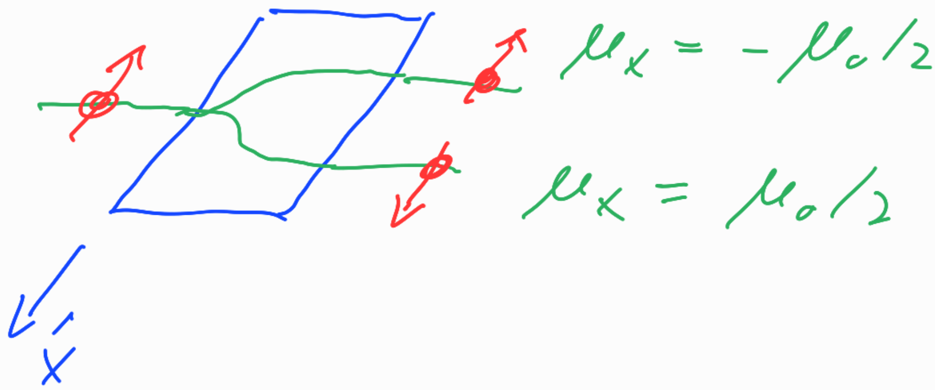


$$\left(\begin{array}{lll} \mu_z = +\mu_0/2 & : & \varphi_{z+} \in \mathcal{H} \\ \mu_z = -\mu_0/2 & : & \varphi_{z-} \in \mathcal{H} \end{array} \right)$$

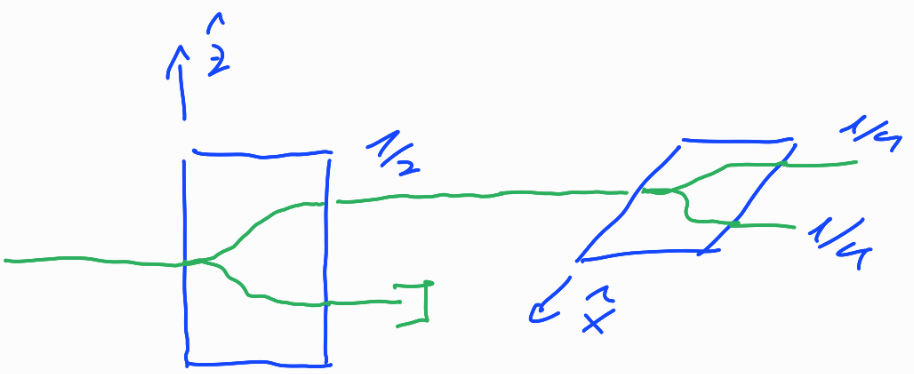
b)



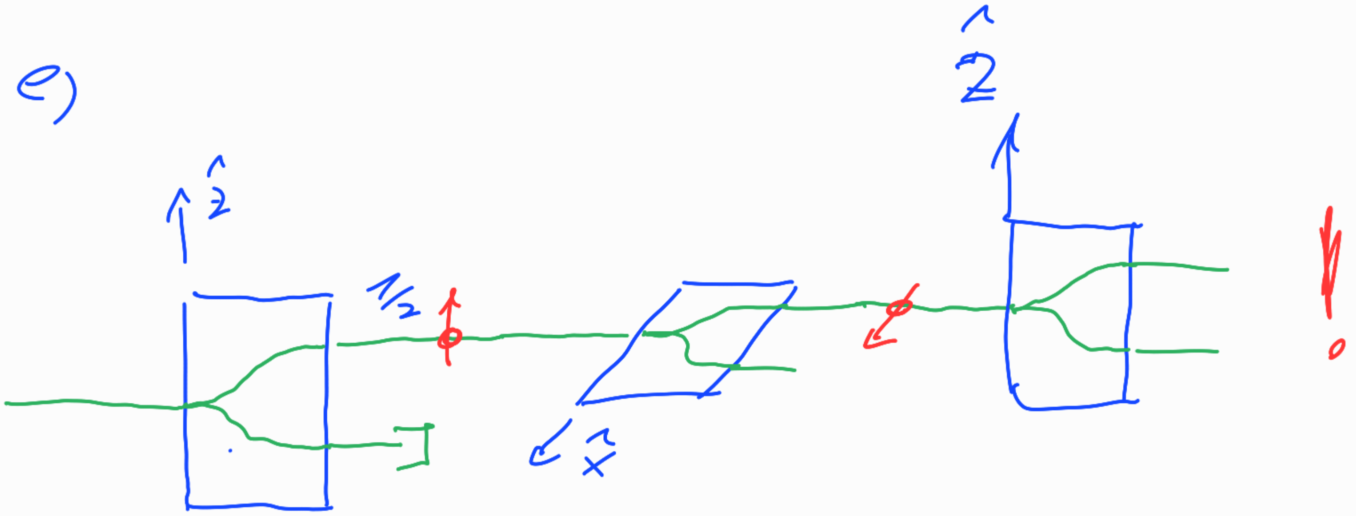
c) Orientierung des S.-G.-Mag. in $\hat{x}$-Richtung:



d)



e)



QM – Beschreibung:

Zustandsraum = $\underline{\underline{2 \text{ dim}}}$ VR $\mathcal{H}$

$\mu_z = +\mu_0/2$: $\varphi_{z+} \in \mathcal{H}$

$\mu_z = -\mu_0/2$: $\varphi_{z-} \in \mathcal{H}$

$\varphi_{z+} \perp \varphi_{z-}$

analog in x-Richtung:

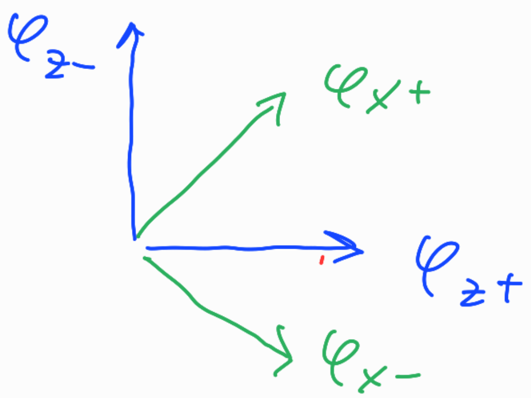
$\mu_x = +\mu_0/2$: $\varphi_{x+} \in \mathcal{H}$

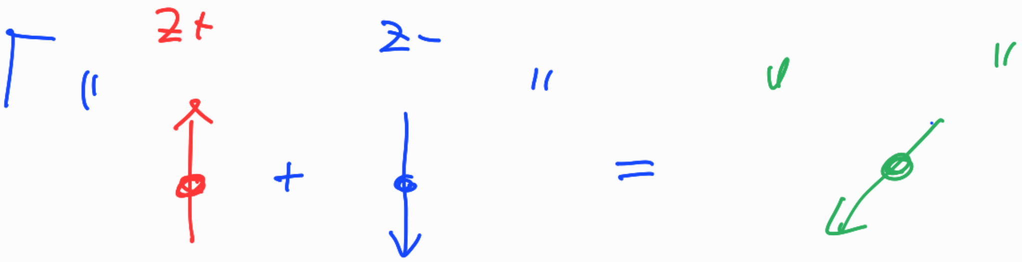
$\mu_y = -\mu_0/2$: $\varphi_{x-} \in \mathcal{H}$

$\varphi_{x+} \perp \varphi_{x-}$

$$\frac{1}{2} \overset{!}{=} p = |\langle \varphi_{x+}, \varphi_{z+} \rangle|^2$$
$$= |\langle \varphi_{x+}, \varphi_{z-} \rangle|^2$$
$$= |\langle \varphi_{x-}, \varphi_{z+} \rangle|^2$$
$$= |\langle \varphi_{x-}, \varphi_{z-} \rangle|^2$$

→ erfüllt für: $\boxed{\varphi_{x\pm} \overset{!}{=} \frac{1}{\sqrt{2}}\left(\varphi_{z+} \pm \varphi_{z-} \right)}$





Polarisierung in y-Richtung (analog!)

$\mu_y = +\mu_0/2$: $\varphi_{y+} = \frac{1}{\sqrt{2}}\left(\varphi_{z+} + i\, \varphi_{z-} \right)$

$\mu_y = -\mu_0/2$: $\varphi_{y-} = \frac{1}{\sqrt{2}}\left(\varphi_{z+} - i\, \varphi_{z-} \right)$