Wdhlg.: <u>Zeitentwicklungsoperator</u>

$$U(t) = e^{-iHt/\hbar}$$

$$\rightsquigarrow \quad |\psi_0\rangle \xrightarrow{t} |\psi(t)\rangle = U(t)|\psi_0\rangle$$

- $U(t)$ <u>unitär</u>: $\quad U(t)^\dagger = U^{-1}(t)$

  (d.h. $U(t)$ Norm-erhaltend, invertierbar)

- $U(t+t') = U(t)U(t')$

<u>Energiedarstellung</u>:

Eigenenergien

Energiezustände

$$H = \sum_n E_n |\varphi_n\rangle\langle\varphi_n|$$

$$\left(H|\varphi_n\rangle = E_n|\varphi_n\rangle \right)$$

$$\downarrow$$

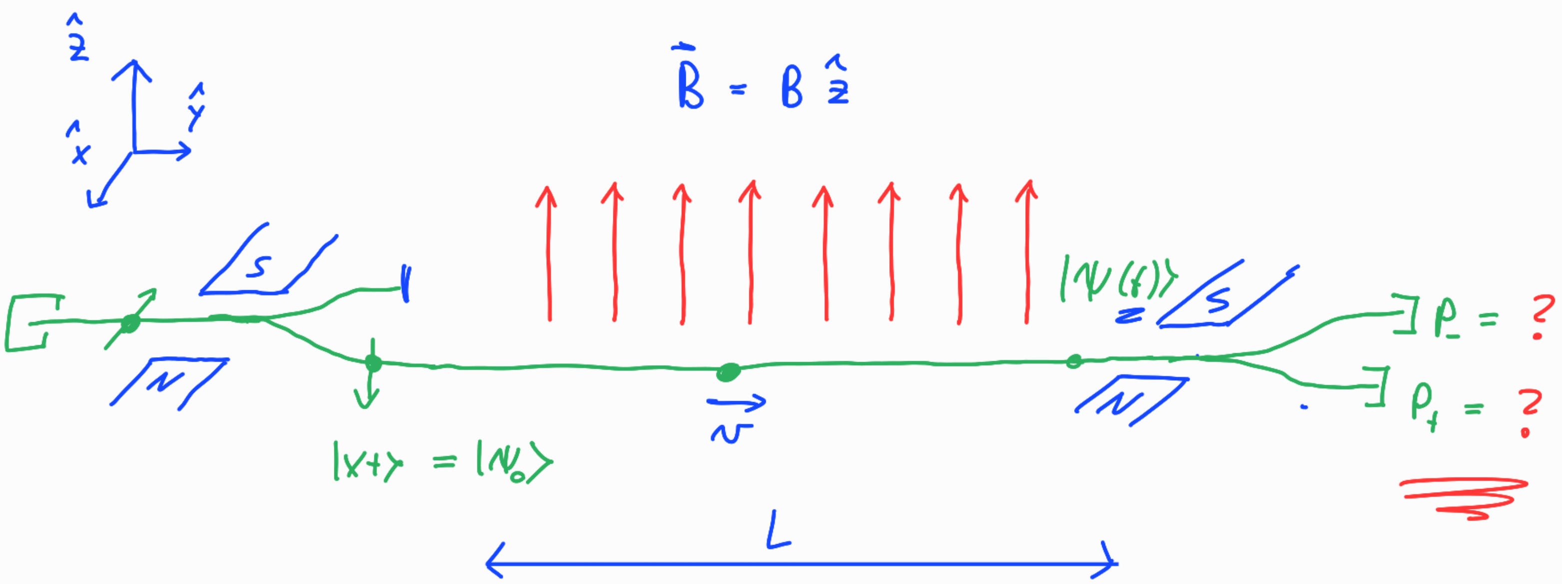
$$U(t) = \sum_n e^{-iE_n t/\hbar} |\varphi_n\rangle\langle\varphi_n| = \sum_n e^{-i\omega_n t} |\varphi_n\rangle\langle\varphi_n|$$

$$\rightsquigarrow \quad |\psi_0\rangle = \sum_k a_k |\varphi_k\rangle \xrightarrow{t} |\psi(t)\rangle = \sum_k e^{-i\omega_k t} a_k |\varphi_k\rangle$$

Beispiel: **Dynamik eines Spins im homog. B-Feld**

$\mu \hat{=}$ mag. Moment

zweistufiges Stern-Gerlach-Experiment:



$t = L/v$

Hamilton-Op.: $H = -B\hat{\mu}_z$

$$= \mu_0 B \left(|z-\rangle\langle z-| - |z+\rangle\langle z+| \right) = \mu_0 B \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

d.h. $|z\pm\rangle$ Energiezust. zu Eigenenergien $\mp \mu_0 B$

$$\rightsquigarrow \quad U(t) = e^{-i\omega t} |z-\rangle\langle z-| + e^{+i\omega t} |z+\rangle\langle z+| \qquad \omega = \frac{\mu_0 B}{\hbar}$$

$$\rightsquigarrow |\psi_0\rangle = |x+\rangle = \frac{|z+\rangle + |z-\rangle}{\sqrt{2}} \xrightarrow{t} \boxed{|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{+i\omega t}|z+\rangle + e^{-i\omega t}|z-\rangle \right)} \rightsquigarrow$$

$$\boxed{|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{+i\omega t}|z+\rangle + e^{-i\omega t}|z-\rangle\right)} = \boxed{\cos(\omega t)\,|x+\rangle + i\sin(\omega t)\,|x-\rangle}$$

$$|z\pm\rangle = \frac{|x+\rangle \pm |x-\rangle}{\sqrt{2}}$$

$$e^{i\omega t} \pm e^{-i\omega t} = 2\cdot\begin{cases}\cos(\omega t)\\ i\sin(\omega t)\end{cases}$$

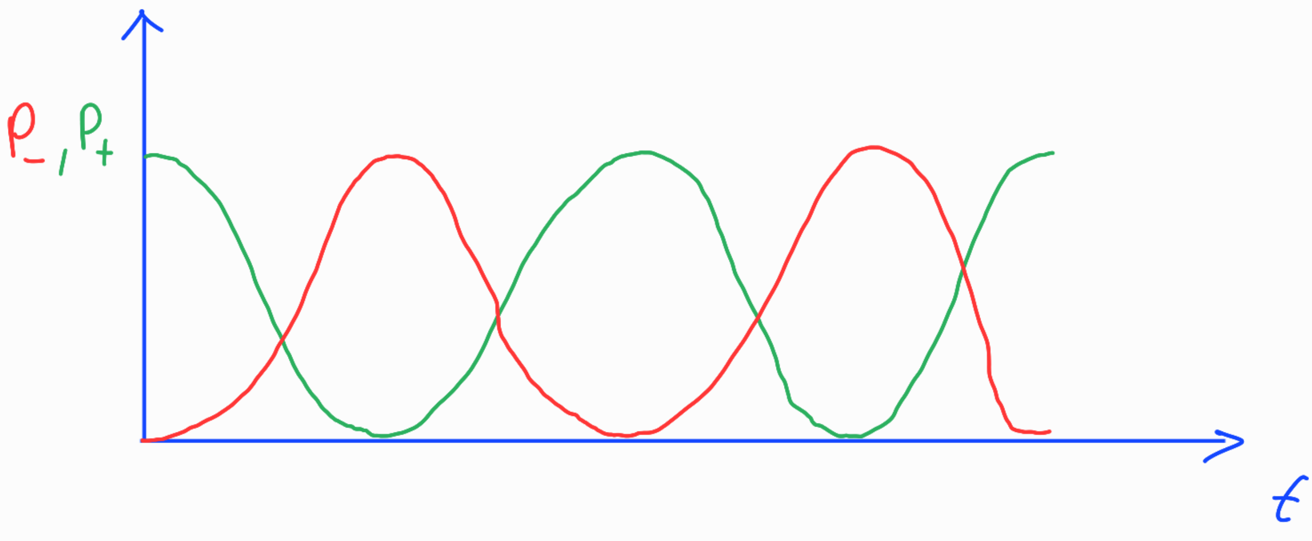
$$\rightarrow \quad P_+(t) = |\langle x+|\psi(t)\rangle|^2 = \cos^2(\omega t)$$

$$P_-(t) = |\langle x-|\psi(t)\rangle|^2 = \sin^2(\omega t) \qquad (= 1 - P_+(t))$$

$$\rightarrow \quad \langle \mu_x\rangle_t = \mu_0\left(P_+(t) - P_-(t)\right) =$$

$$= \mu_0\left(\cos^2(\omega t) - \sin^2(\omega t)\right)$$

$$= \mu_0 \cos(2\omega t)$$



alternative Rechnungen:

b) $U(t) = e^{i\omega t} |z+\rangle\langle z+| + e^{-i\omega t} |z-\rangle\langle z-| = \begin{pmatrix} e^{i\omega t} & 0 \\ 0 & e^{-i\omega t} \end{pmatrix}$

$\rightarrow \quad P_+(t) = |\langle x+ | \psi(t)\rangle|^2 = |\langle x+ | U(t) | x+\rangle|^2$

$$= \left| \frac{1}{2}(1,1) \begin{pmatrix} e^{i\omega t} & 0 \\ 0 & e^{-i\omega t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right|^2$$

$$= \left| \frac{1}{2}(1,1) \begin{pmatrix} e^{i\omega t} \\ e^{-i\omega t} \end{pmatrix} \right|^2 = \left| \frac{1}{2}(e^{i\omega t} + e^{-i\omega t}) \right|^2$$

$$= \cos^2(\omega t)$$

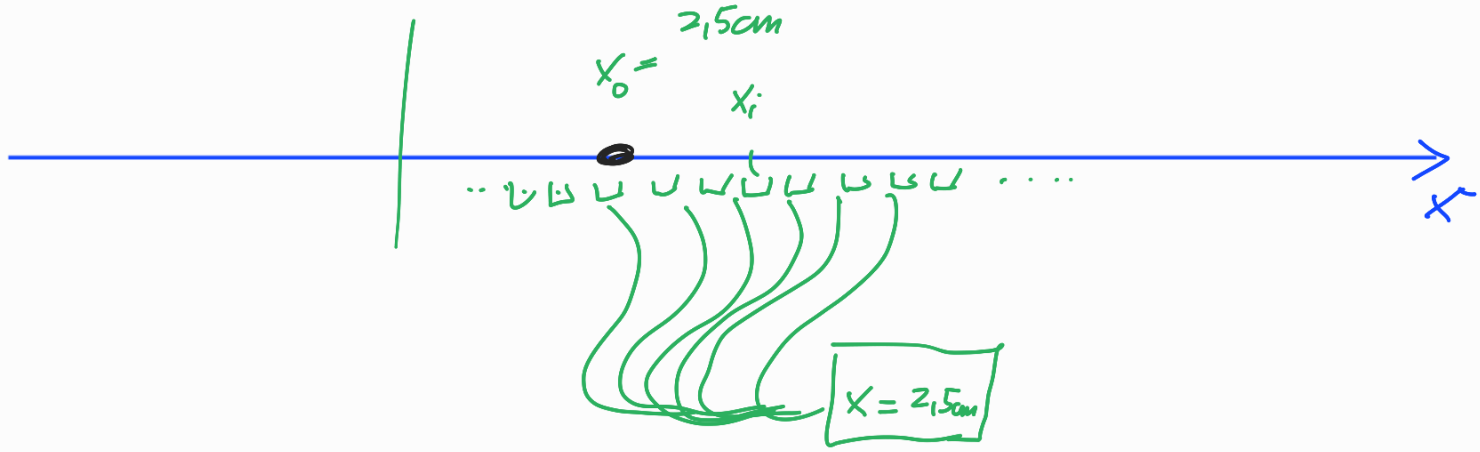
1) mittels Pauli-Operatoren:

$$H = -B\hat{\mu}_z = -B\mu_0 \sigma_3 \quad \left(= -B\mu_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right)$$

$$\rightarrow U(t) = e^{i \underbrace{\frac{B\mu_0 t}{\hbar}}_{\overset{!!}{=}\,\omega t} \cdot \sigma_3} \underset{\text{Euler-Formel}}{=} \cos(\omega t) \underset{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}{\mathbb{1}_2} + i \sin(\omega t) \overset{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}{\sigma_3}$$

$$= \begin{pmatrix} e^{i\omega t} & 0 \\ 0 & e^{-i\omega t} \end{pmatrix}$$ dann wie in b) ....

Quantenmechanik des Punktteilchen (1D)



Ortsmessung: Resultat $x \in \mathbb{R}$

↳ Ortsobservable $\hat{x}$ = herm. Operator

mit Kontinuum an EWen $\{x\}_{x \in \mathbb{R}}$

mit zugeh. EZen $\{ |\varphi_x\rangle \}_{x \in \mathbb{R}}$

→ Anpassung des Dirac-Formalismus

diskret (D) kontinuierlich (K)

D $\qquad \sum_i \cdots \longrightarrow \int_{-\infty}^{\infty} dx \cdots \qquad$ K

Obs. $A \hat{=} \hat{A} = \sum_i a_i |\varphi_i\rangle\langle\varphi_i|$

EWe $\{a_i\}_{i=1,2,3,\dots}$

<u>Eze</u> $\{|\varphi_i\rangle\}_{i=1,2,3,\dots}$

ONB!

Ortsob. x = Ortsop. $\hat{X}$

$$\hat{X} = \int_{-\infty}^{\infty} dx\; x\, |\varphi_x\rangle\langle\varphi_x|$$

EWe $\{x\}_{x\in\mathbb{R}}$

Eze $\{|\varphi_x\rangle\}_{x\in\mathbb{R}}$

Orthonormalität

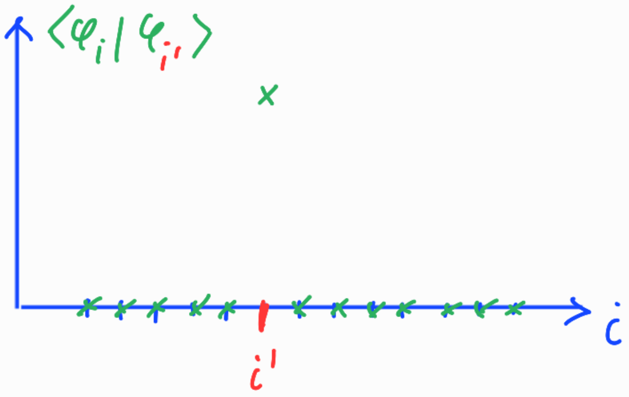
der $\{|\varphi_i\rangle\}$:

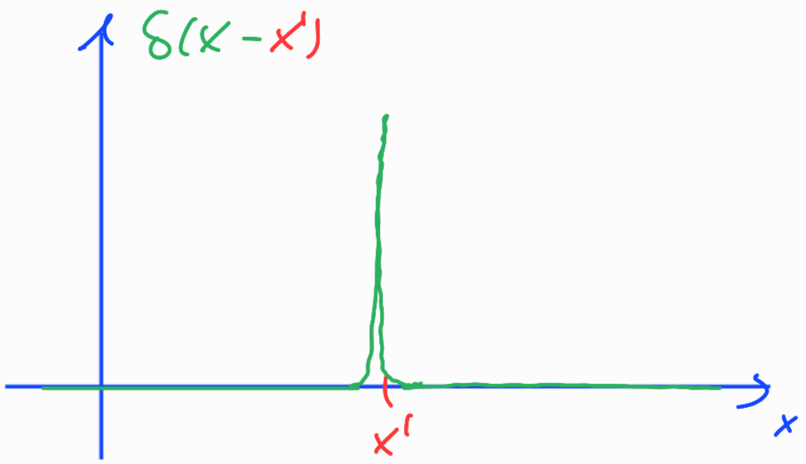
$$\langle\varphi_i|\varphi_{i'}\rangle = \delta_{ii'}$$

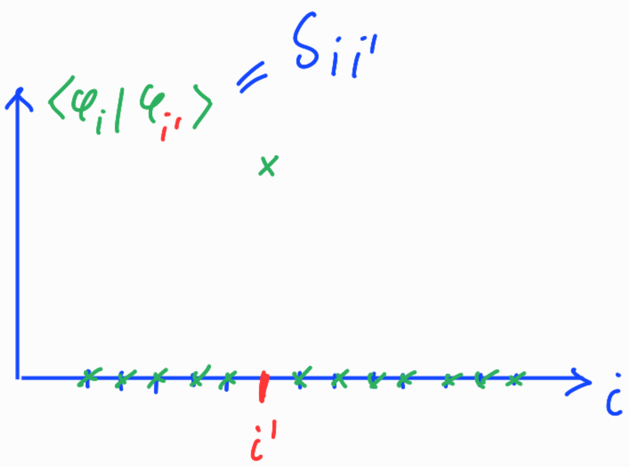
der Ortszust. $\{|\varphi_x\rangle\}_{x\in\mathbb{R}}$

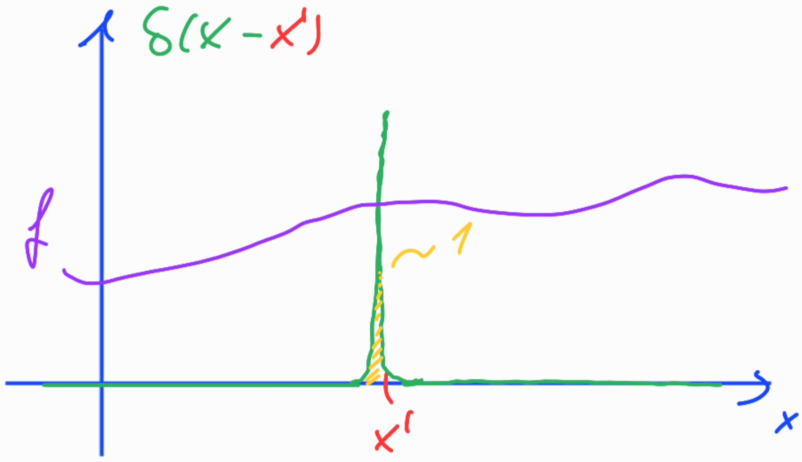
$$\langle\varphi_x|\varphi_{x'}\rangle = \delta(x-x')$$

Dirac-Delta-Fkt.









- $i \neq i'$: $\delta_{ii'} = 0$
- $\sum_i \delta_{ii'} = 1$
- $\sum_i \delta_{ii'} \cdot f_i = f_i$

$$\boxed{\begin{array}{l} \bullet\ x \neq x' : \quad \delta(x-x') = 0 \\ \bullet\ \int_{\mathbb{R}} dx\, \delta(x-x') = 1 \\ \bullet\ \int_{\mathbb{R}} dx\, \delta(x-x') f(x) = f(x') \end{array}}$$

↑ definierende Eigenschaften

die δ-Fkt ≡ Rechenregeln!

Vollständigkeit

der Zust. $\{|\varphi_i\rangle\}_{i=1,2,\dots}$

$$\boxed{\mathbb{1}_{\mathcal{H}} = \sum_i |\varphi_i\rangle\langle\varphi_i|}$$

↳ Komponentendarstellung des Zustands $|\psi\rangle$:

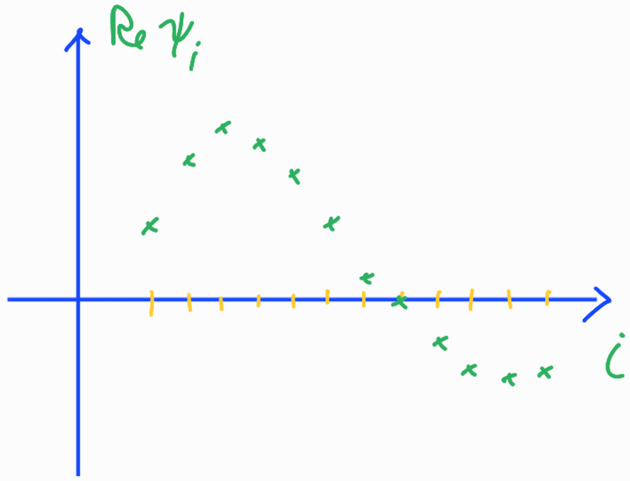
$$|\psi\rangle = \mathbb{1}_{\mathcal{H}} |\psi\rangle$$

$$= \Big(\sum_i |\varphi_i\rangle\langle\varphi_i|\Big)|\psi\rangle$$

$$= \sum_i \langle\varphi_i|\psi\rangle\, |\varphi_i\rangle$$

$$|\psi\rangle = \sum_i \psi_i\, |\varphi_i\rangle$$

ψ_i: i-te Komponente von $|\psi\rangle$



der Zust $\{|\varphi_x\rangle\}_{x\in\mathbb{R}}$

$$\boxed{\mathbb{1}_{\mathcal{H}} = \int_{\mathbb{R}} dx\, |\varphi_x\rangle\langle\varphi_x|}$$

↳ Ortsdarstellung des Zustands $|\psi\rangle$

$$|\psi\rangle = \mathbb{1}_{\mathcal{H}} |\psi\rangle$$

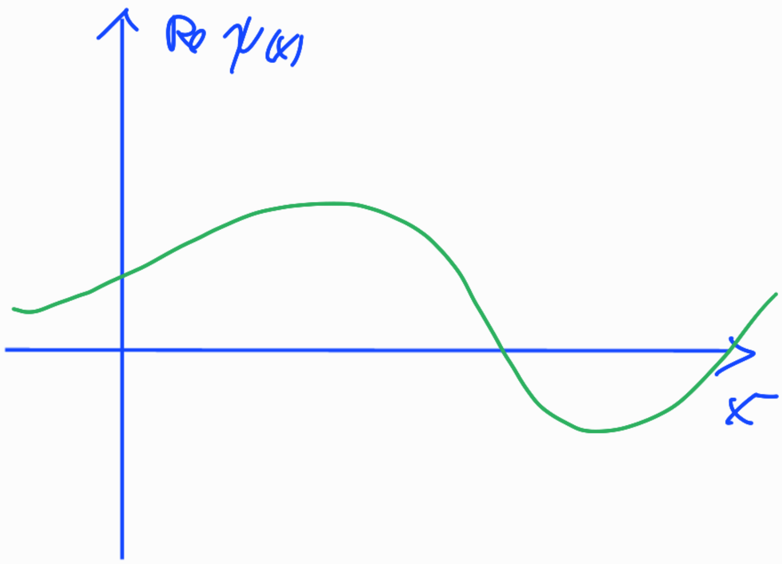
$$= \Big(\int_{\mathbb{R}} dx\, |\varphi_x\rangle\langle\varphi_x|\Big)|\psi\rangle$$

$$= \int_{\mathbb{R}} dx\, |\varphi_x\rangle \underbrace{\langle\varphi_x|\psi\rangle}_{\psi(x)}$$

mit Wellenfunktion des Zustands $|\psi\rangle$ (am Ort x):

$$\psi(x) := \langle\varphi_x|\psi\rangle$$

$$|\psi\rangle = \int_{\mathbb{R}} dx\, \psi(x)\, |\varphi_x\rangle$$



Skalarprodukt

$$\langle \psi | \chi \rangle = \left(\sum_i \psi_i^* \langle \varphi_i | \right) \left(\sum_j \chi_j | \varphi_j \rangle \right)$$

$$= \sum_{ij} \psi_i^* \chi_j \underbrace{\langle \varphi_i | \varphi_j \rangle}_{\delta_{ij}}$$

$$= \sum_i \psi_i^* \chi_i$$

$$\langle \psi | \chi \rangle = \sum_i \psi_i^* \chi_i$$

$$\langle \psi | \chi \rangle = \left(\int dx \, \psi^*(x) \langle \varphi_x | \right) \cdot \left(\int dx' \, \chi(x') | \varphi_{x'} \rangle \right)$$

$$= \int dx \int dx' \, \psi^*(x) \chi(x') \underbrace{\langle \varphi_x | \varphi_{x'} \rangle}_{= \delta(x - x')}$$

$$= \int dx \, \psi^*(x) \chi(x)$$

$$\boxed{\langle \psi | \chi \rangle = \int dx \, \psi^*(x) \chi(x)}$$