

## Anwendungsbeispiele:

1) Berechnung von  $I(h, a) := \int_{-\infty}^{\infty} \frac{e^{-ikhx}}{x^2 + a^2} dx$ ,  $h \in \mathbb{R}$   
 $a \in \mathbb{R}_+$

fasse  $I(h, a)$  als kompl. Kurvenintegral über reelle Achse  $\mathbb{R} \subset \mathbb{C}$   
auf:

$$I(h, a) = \int_{\mathbb{R}} \underbrace{\frac{e^{-ikhz}}{(z+ia)(z-ia)}}_{\text{meromorph}}$$

Pole 1. Ordnung bei  $\mp ia$

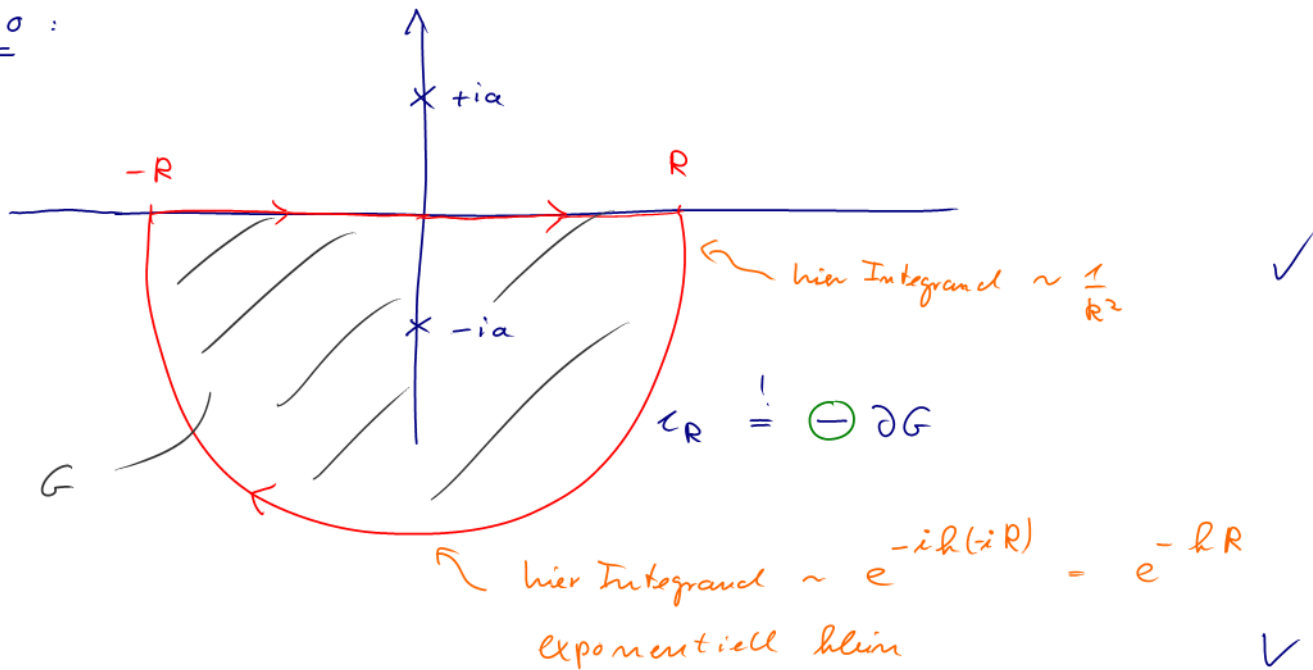
Idee: finde geschlossenen Weg  $\gamma_R \subset \mathbb{C}$  so, dass

$$I(h, a) \stackrel{!}{=} \lim_{R \rightarrow \infty} \int_{\gamma_R} \frac{e^{-ikhz}}{(z+ia)(z-ia)} dz$$

mittels Residuensatz  
bestimmbar



sei  $h > 0$  :



$$\begin{aligned} \Rightarrow I(h, a) &\stackrel{!}{=} \lim_{R \rightarrow \infty} \int_{C_R} \frac{e^{-hz}}{(z-ia)(z+ia)} dz = \ominus 2\pi i \operatorname{Res}(\dots, -ia) \\ &= -2\pi i \frac{e^{-ha}}{-2ia} = \frac{\pi}{a} e^{-ha} \end{aligned}$$

Falls  $h < 0$  schließt man Weg in oberen Halbebene

$$\hookrightarrow I(h, a) = \frac{\pi}{a} e^{+ha} ;$$

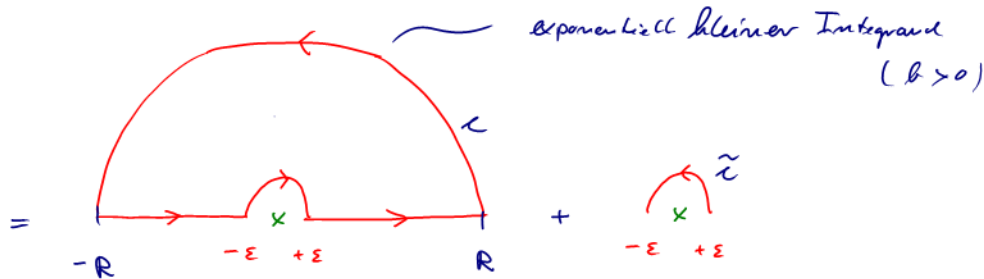
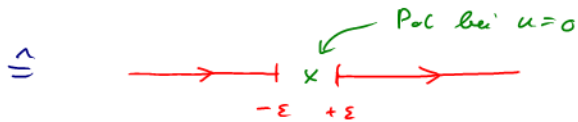
d.h.

$$I(h, a) \equiv \int_{-a}^a \frac{e^{-ihx}}{x^2 + a^2} dx = \frac{\pi}{a} e^{-|h|a}$$

$$2) \quad \mathcal{J}(b) := \int_{-\infty}^{\infty} du \frac{e^{ibu}}{u} \equiv \lim_{\varepsilon \rightarrow 0^+} \left( \int_{-\infty}^{-\varepsilon} du \frac{e^{ibu}}{u} + \int_{+\varepsilon}^{+\infty} du \frac{e^{ibu}}{u} \right)$$

$\uparrow$   
 "Hauptwert" des Integrals

$(b > 0)$



$$\tilde{\gamma} : [0, \pi] \rightarrow \mathbb{C}$$

$$t \mapsto \varepsilon e^{it}$$

$$= \int_{\mathcal{L}_R} \frac{e^{ibz}}{z} dz + \int_0^{\pi} \frac{e^{ib\varepsilon e^{it}}}{\varepsilon e^{it}} i \varepsilon e^{it} dt = \underline{\underline{\pi i}}$$

$\parallel$   
 $0$

$\hookrightarrow 1$   
 $\varepsilon \rightarrow 0$

Falls  $b < 0$  muss in unterer Halbebene geschlossen werden:

$$J(b) = \text{Diagram} + \text{Diagram} = -\pi i$$

The diagram shows a contour in the lower half-plane. It consists of a large semi-circle in the lower half-plane with arrows pointing clockwise. Above it, there is a horizontal line segment with arrows pointing to the right. A small semi-circle with an 'x' above it and an arrow pointing clockwise is drawn below the horizontal line segment, indicating a pole at the origin. To the right of the main diagram, there is a smaller diagram of a semi-circle with an arrow pointing clockwise, representing the contribution from the pole at the origin.

d.h.

$$J(b) \equiv \int_{-a}^a \frac{e^{ibu}}{u} du = \pi i \operatorname{sgn} b$$