Lösungen zu Blatt 4

1.) Heizleistung

$$U(t) = U_0 \cos(2\pi f t)$$

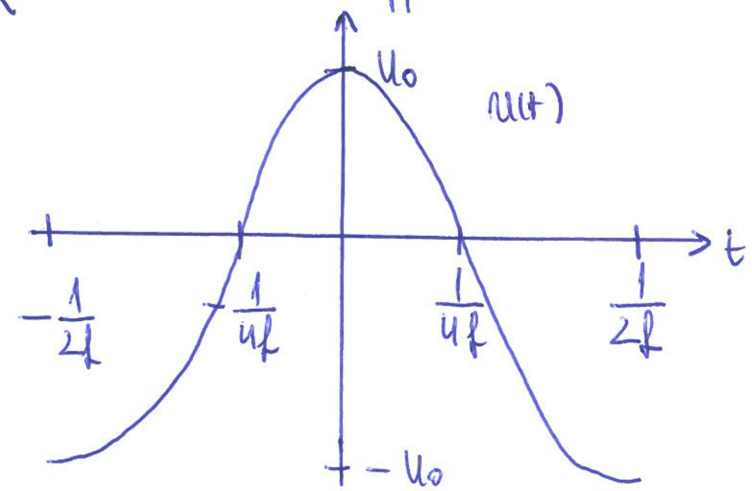
$$U^2(t) = U_0^2 \cos^2(2\pi f t)$$

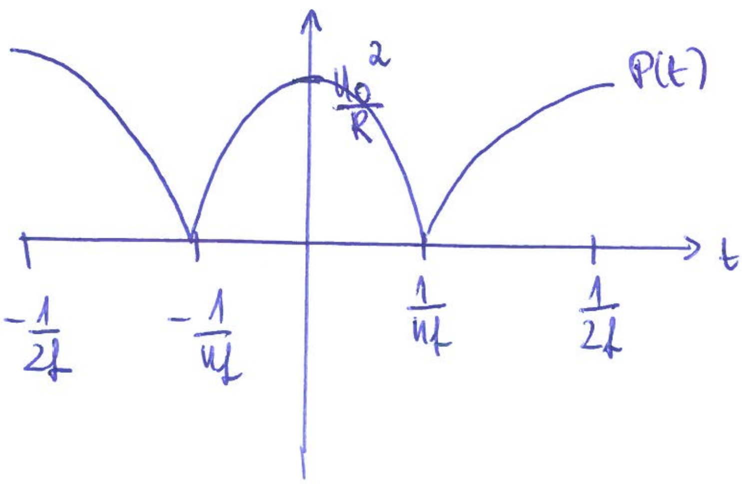
$$= U_0^2 \left[\cos(4\pi f t) + \sin^2(2\pi f t) \right]$$

auch periodisch mit $\tilde{T}$

wenn $f = 1/T$, dann ist dieser Term periodisch mit $\tilde{T} = \frac{T}{2}$

$\Rightarrow P(t) = \frac{U^2(t)}{R}$ oszilliert doppelt so schnell





2) Additionstheoreme

Sei $\alpha > \beta$, $0 \leq \alpha - \beta \leq \pi$

$$\langle \hat{a}, \hat{b} \rangle = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)$$

$$\hat{a} \times \hat{b} = \begin{pmatrix} 0 \\ 0 \\ \cos\alpha\sin\beta - \sin\alpha\cos\beta \end{pmatrix}$$

$$\Rightarrow |\hat{a} \times \hat{b}| = |\cos\alpha\sin\beta - \sin\alpha\cos\beta| = \sin(\alpha-\beta)$$

Da $0 \leq \alpha - \beta \leq \pi$ ist $\sin(\alpha-\beta) > 0$

$$\Rightarrow \cos\alpha\sin\beta - \sin\alpha\cos\beta = \sin(\alpha-\beta)$$

3) Sekanten und Tangentensteigung

$f(x) = x^3 - 3x$

Sekantensteigung: $m = \dfrac{x_2^3 - 3x_2 - x_1^3 + 3x_1}{x_2 - x_1}$

$= 1.75$

$f'(x) = 3x^2 - 3$

Tangentensteigung: $f'(x_1) = 3 \cdot 1^2 - 3 = 0$

$f'(x_2) = 3 \cdot 1{,}5^2 - 3 = 3.75$

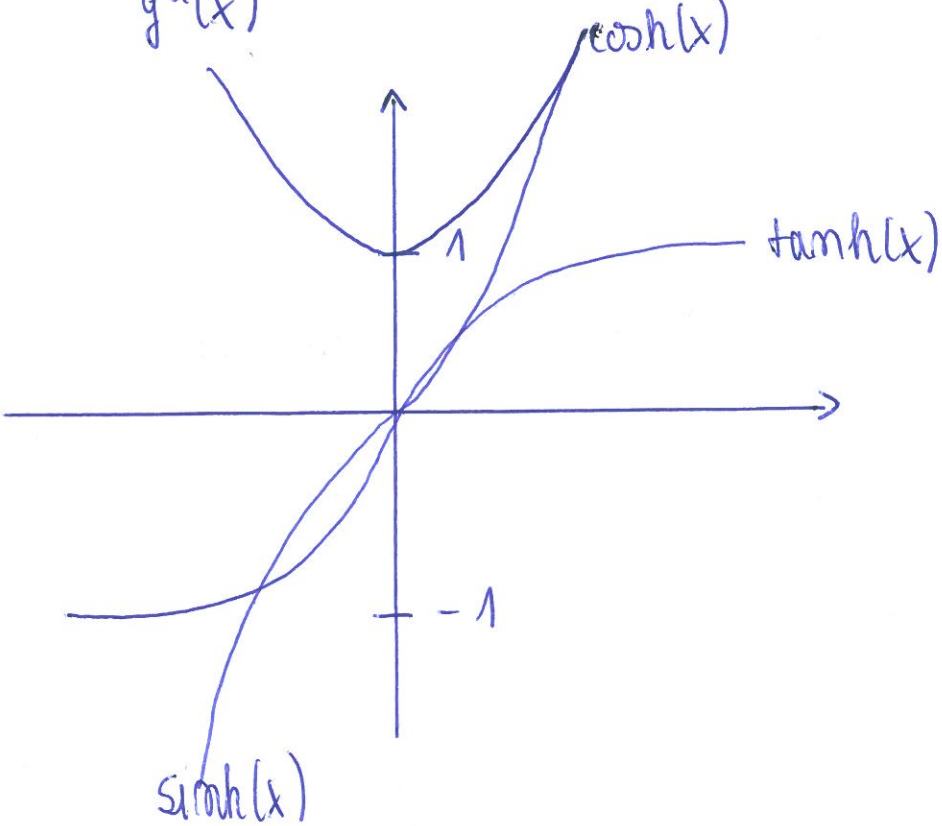
4.) Hyperbolische Funktionen und Quotientenregel

1)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{d}{dx}\left[f(x)\cdot(g(x))^{-1}\right] = \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{g^2(x)}$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

2)



3) sinh(x) und tanh(x) sind ungerade
cosh(x) ist gerade

4) $\cosh^2(x) = \frac{1}{4}(e^x + e^{-x})^2 = \frac{1}{4}\left[e^{2x} + 2 + e^{-2x}\right]$

$\sinh^2(x) = \frac{1}{4}(e^x - e^{-x})^2 = \frac{1}{4}\left[e^{2x} - 2 + e^{-2x}\right]$

$\cosh^2(x) - \sinh^2(x) = \frac{1}{2} + \frac{1}{2} = 1$

5)

$$\frac{d}{dx}\sinh(x) = \frac{1}{2}\frac{d}{dx}\left(e^{x} - e^{-x}\right)$$

$$= \frac{1}{2}\left(e^{x} + e^{-x}\right) = \cosh(x)$$

$$\frac{d}{dx}\cosh(x) = \sinh(x)$$

$$\Rightarrow \frac{d^2}{dx^2}\sinh^2(x) = \sinh(x), \quad \frac{d^2}{dx^2}\cosh^2(x) = \cosh(x)$$

6)

$$\frac{d}{dx}\tanh(x) = \frac{d}{dx}\left(\frac{\sinh(x)}{\cosh(x)}\right) \overset{a)}{=} \frac{\cosh^2(x) - \sinh^2(x)}{\cosh(x)^2}$$

$$= \frac{1}{\cosh^2(x)}$$

5) Ableitungen

a) $y = \dfrac{x^3 - x}{6x^2}$, $y' = \dfrac{(3x^2-1)6x^2 - (x^3 - x)\cdot 12x}{36x^4}$

$$= \frac{18x^4 - 6x^2 - 12x^4 + 12x^2}{36x^4}$$

$$= \frac{6x^4 + 6x^2}{36x^4} = \frac{x^4 + x^2}{6x^4}$$

b) $y = \ln\left(\sqrt{x^2+1}\right)$, $y' = \dfrac{1}{\sqrt{x^2+1}}\,\dfrac{1}{2\sqrt{x^2+1}}\cdot 2x$

$$= \frac{x}{x^2+1}$$

c) $y = x e^{x} \sin(x)$

$$y' = \left[e^{x} + x e^{x} \right] \sin(x) + x e^{x} \cos(x)$$

$$= e^{x} \left(x \left(\sin(x) + \cos(x) \right) + \sin(x) \right)$$

d) $y = \log_{10}(x)$, $y' = \left(\dfrac{\ln(x)}{\ln(10)} \right)'$

$$= \frac{1}{\ln(10) \cdot x}$$