Information Theory & Statistical Physics

Lecture: Johannes Berg
Exercises: Stephan Kleinbölting

Sheet 6

Due date: 20.07.17 12:00 *To be discussed on: 26.07.17*

Website: http://www.thp.uni-koeln.de/~skleinbo/teaching/Information2017/

In this sheet we implement a belief propagation (BP) algorithm on systems of Ising spins.

18 Belief propagation in the Ising model 60 pts.

In the lecture the belief propagation update rules for a binary spin system with Hamiltonian

$$-\sum_{i<j} J_{ij}\sigma_i\sigma_j - \sum_i h_i\sigma_i \tag{1}$$

were deduced.

We want to use belief propagation to find the critical temperature of the 2d Ising model on a square lattice in the absence of an external field $h_i = 0$.

Given a coupling matrix J_{ij} the message between sites are updated according to

$$H_{i\to j} = \frac{1}{\beta}\,\mathrm{atanh}\left\{\tanh(\beta J_{ij})\tanh(\beta\sum_k H_{k\to i})\right\} \tag{2}$$

The sum extends over messages from neighbouring spins of i *that are not* j: $k \in \mathcal{N}(i)\setminus j$.

(a) *10pt* - By definition

$$H_{i\to j} = \frac{1}{2\beta}\log\frac{r_{i\to j}(+)}{r_{i\to j}(-)}. \tag{3}$$

On the other hand the (not normalized) marginals $p_j^*(\sigma_j)$ are computed as

$$p_j^*(\sigma_j) = \prod_{k\in\mathcal{N}(j)} r_{k\to j}(\sigma_j). \tag{4}$$

Combine eq. (3) and (4) to show that the magnetization at site j is given by

$$m_j = \tanh(\beta\sum_{k\in\mathcal{N}(j)} H_{k\to j}) \equiv \tanh(\beta H_j). \tag{5}$$

(b) *50pt* - Implement the algorithm in your favorite programming language for a general coupling matrix!

Run your program on a 2d square lattice with homogeneous couplings $J_{ij} = J = 1$ for an appropriate range of temperatures. You may use open boundary conditions. Think about how to initialise the messages.

Find the magnetization and plot the resulting curve. Determine the critical temperature and compare to the analytical solution.